September 29, 2006 **Math 121B** Name Key - solutions

Instructor: Chávez-Ross or Mahaffy Lab Section time ______

THEORETICAL EXAM

Note there are FOUR pages in this exam.
Give all answers to at least **4 significant figures.**

1. **(25 pts)** Consider the functions:

$$f(x) = 2x - 8 \qquad \text{and} \qquad g(x) = 4x - x^2.$$

Find the coordinates of the x and y-intercepts for both functions. Find the slope of the line and the coordinates for the vertex of the parabola. Determine the coordinates for all points of intersection and sketch the graph.

For the line:

∘ x-intercept (,) ∘ y-intercept (,) ∘ slope $m =$ ______

For the parabola:

∘ x-intercept (,) and (,)

∘ y-intercept (,)

∘ vertex (______ , ______).

Points of intersection:

∘ $(x, y) =$ (______ , ______ , and (______ , ______).

2. **(15 pts)** For the following function, find the domain. Find all x and y-intercepts. Determine any horizontal or vertical asymptotes. (If no asymptote exists, then write "NONE."). Sketch the graph of the function.

$$y = \frac{4x^2}{x^2 - 4}.$$

∘ Domain: ______

∘ x-intercept ______

∘ y-intercept ______

∘ Vertical Asymptotes: ______

∘ Horizontal Asymptotes: ______

3. **(30 pts)** In lab, we saw that the early growth phase of yeast (and many other organisms) follow the dynamics of a discrete Malthusian growth model:

$$P_{n+1} = P_n + r\,P_n,$$

where the term $r\,P_n$ represents a growth proportional to current population. Define the linear growth function

$$G(P) = r\,P,$$

where growth is proportional to the population present. Below are data collected on the growth of several populations of a bacteria in culture.

P (×1000)	1	3	4
$G(P)$ (×1000/hr)	0.3	0.8	1.5

a. The data above are used to find the best slope r for the growth function $G(P)$. Write all the square errors. Write a quadratic function $J(r)$ that measures the sum of squares error based on the standards above for the line fitting the data. Find the vertex of this quadratic function. This gives the value of the best slope r, while the $J(r)$ value of the vertex gives the least sum of squares error.

- $e_1^2 =$

  $e_2^2 =$ ______

  $e_3^2 =$ ______

- $J(r) =$ ______ $r^2 +$ ______ $r +$

- The vertex location $r_v =$ ______

- The least sum of squares error $J(r_v) =$

b. Use this model (with the best value of r) to predict the rate of growth of the population when $P = 2$ (×1000). Also, find the current population P if you measure a growth rate $G(P) = 0.6$ (×1000/hr).

4. **(30 pts)** Many enzymes follow Michaelis-Menten kinetics and have the following form for their reaction rates:

$$R([S]) = \frac{V_{max}[S]}{K_m + [S]},$$

where $[S]$ is the concentration of the substrate that the enzyme is catalyzing, $R([S])$ is the rate of production of the product, V_{max} is the maximum rate of the reaction.

a. Suppose that a particular enzyme is found to $V_{max} = 400$ nM/sec and $K_m = 250$ μM, so

$$R([S]) = \frac{400[S]}{250 + [S]}.$$

Find the $[S]$ and $R([S])$ intercepts and any asymptotes for $[S] \geq 0$. (If no asymptote exists in the domain, then write "NONE."). Sketch a graph of $R([S])$ for $[S] \geq 0$.

○ $[S]$-intercept _

○ $R([S])$-intercept

○ Vertical Asymptotes: ________

○ Horizontal Asymptotes: _

○ **GRAPH:**

b. An easy method for finding the constants V_{max} and K_m is using the Lineweaver-Burk line. The enzyme satisfies the linear equation

$$y = \frac{K_m}{V_{max}}x + \frac{1}{V_{max}},$$

where $x = 1/[S]$ and $y = 1/R([S])$. Suppose that a related enzyme to the one above is found to have a reaction rate, $R = 16$ nM/sec, when $[S] = 10$ μM, and it has a reaction rate of $R = 100$ nM/sec, when $[S] = 80$ μM. Use this information to find the Lineweaver-Burk line, then compute V_{max} and K_m.

○ Equation of line

○ $V_{max} =$ ___

○ $K_m =$

Fall 2006 Math 121B Name Key-solution

Instructor (Chávez-Ross or Mahaffy - circle) Lab Section ____________

Give all answers to at least **4 significant figures.**

1. We studied enzymes that follow Michaelis-Menten kinetics and have the following form for their reaction rates:

$$R([S]) = \frac{V_{max}[S]}{K_m + [S]},$$

where $[S]$ is the concentration of the substrate that the enzyme is catalyzing, $R([S])$ is the rate of production of the product, V_{max} is the maximum rate of the reaction (and depends on the enzyme concentration used), and K_m is the Michaelis-Menten constant, which characterizes a particular enzyme.

Kinesin is an important motor protein walking along microtubules using ATP. Below are data from Hancock and Howard [1] for ATPase activities associated with one-headed kinesin as concentrations of tubulin are varied. Their work helps understanding this important dynamic process for movement within cells.

[Tubulin] (μM)	ATPase rate (s^{-1})
0.1	0.73
0.5	1.82
1.0	2.17
2.0	2.46
3.0	2.58
5.0	2.67
10.0	2.75

From this table, we let $[S]$ = [Tubulin] and $R([S])$ = ATPase rate. We want to find the best values of V_{max} and K_m to characterize the ATPase rate of one-headed kinesin as tubulin concentration varies.

a. One of the easiest methods for finding the constants V_{max} and K_m is using the Lineweaver-Burk plot as discussed in lecture. To do this, you take the data and create a new table with the values $x = 1/[S]$ and $y = 1/R([S])$. Create this new table in Excel, then plot the data y vs x. Use Trendline to find the best fit to this straight line. The y-intercept has the value $1/V_{max}$ and the slope has the value K_m/V_{max}. Use this information to find V_{max} and K_m.

- Trendline best line is _
- V_{max} = _____
- K_m = _______

b. Write the Michaelis-Menten reaction formula for $[S]$ and $R([S])$ with the constants that you have found. Find all intercepts and any asymptotes for this function for $[S] \geq 0$. (If no asymptote exists, then write "NONE.")

○ MM reaction formula $R([S]) =$ _

○ $[S]$-intercept and R-intercept

○ Vertical Asymptote:

○ Horizontal Asymptote: ________

c. With the Michaelis-Menten reaction formula, find the predicted production rate $R([S])$ when the concentration of tubulin $[S] = 5.0$. Calculate the percent error between the model and the actual data at $[S] = 5.0$. (Use the data as most accurate.)

○ $R([5]) =$ ________

○ Percent Error = ________

2. This problem is probably best done using Maple. Consider the quadratic and rational functions

$$f(x) = 8 - 3x - x^2 \qquad \text{and} \qquad g(x) = \frac{12x}{1.6 - 2x}.$$

(You will probably want to graph these functions on the interval $x \in [-10, 10]$ with the range restricted to $y \in [-50, 50]$.)

Find the x and y-intercepts for both of these functions. Find the vertex of the quadratic function, $f(x)$. Give both x and y values. List any asymptotes (vertical and horizontal) for the rational function, $g(x)$. Finally, find all points of intersection between the graphs of $f(x)$ and $g(x)$.

For the **Quadratic Function**:

○ x-intercepts ___ ________ and y-intercept

○ The vertex $(x_v, y_v) =$ ____

For the **Rational Function**:

○ x-intercept _ and y-intercept _

○ Vertical Asymptote: ___

○ Horizontal Asymptote: _

List all Points of intersection for $f(x)$ and $g(x)$,

○ $(x_p, y_p) =$

[1] Hancock, W. O. and Howard, J. (1999) Kinesin's processivity results from mechanical and chemical coordination between the ATP hydrolysis cycles of the two motor domains, *PNAS* **96**, 13147-13152.