Math 254 Practice Test 2

1. Find all values of k that make $\mathbf{u} = (k, 2, 0, 2-k)$ and $\mathbf{v} = (6, k, k^3, 2-k)$ orthogonal.

2. Find the standard matrix for the following composition of linear transformations that map $\mathbb{R}^2 \to \mathbb{R}^2$:
A reflection across the x-axis followed by a 45° rotation followed by another reflection across the x-axis.

3. A linear transformation $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^2$ sends $\mathbf{e}_1$ to $2\mathbf{e}_1 - \mathbf{e}_2$ and $\mathbf{e}_2$ to $-3\mathbf{e}_1$. What is the standard matrix of $\mathbf{T}$?

4. Consider the vector space $\mathbb{P}(3)$ of cubic polynomials and the linear transformation D: $\mathbb{P}(3) \to \mathbb{P}(3)$, which sends any polynomial to its derivative,

$$D(a + bx + cx^2 + dx^3) = b + 2cx + 3dx^2.$$

Using the standard basis $\{1, x, x^2, x^3\}$ in $\mathbb{P}(3)$, find the standard matrix of the linear transformation D.

5. Find a basis for span($\{(2, 2, 7), (5, 5, 0), (-2, -2, 6)\}$.

6. A 10 by 4 matrix M has rank 3. Find the nullity of M and M^T.

7. Find the rank and nullity of

$$\begin{pmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{pmatrix}$$

8. Find bases for the row space and column space of the matrix in the previous problem.

9. Find a basis for the null space of the matrix in the previous two problems.

10. Express the vector $\mathbf{u} = (3, 5, 7)$ relative to the basis $S\{(1, 3, 5), (0, 0, 1), (1, 1, 1)\}$.

$$(\mathbf{u})_S =$$