1

1.6

1) $x_1 + x_2 = 2$

$5x_1 + 6x_2 = 9$

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}}$$

5) $x + y + z = 5$

$x + y - 4z = 10$

$-4x + y + z = 0$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -4 \\ -4 & 1 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$Ax = B$$

$$\therefore x = A^{-1}B$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}}$$

16) $6x_1 - 4x_2 = b_1$
$3x_1 - 2x_2 = b_2$

Augmented Matrix $\begin{bmatrix} 6 & -4 & b_1 \\ \cancel{6}3 & -\cancel{8}2 & b_2 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & b_1 - \cancel{b}2b_2 \\ 3 & -2 & b_2 \end{bmatrix}$

$\boxed{b_1 - 2b_2 = 0}$

17) $x_1 - 2x_2 + 5x_3 = b_1$
$4x_1 - 5x_2 + 8x_3 = b_2$
$-3x_1 + 3x_2 - 3x_3 = b_3$

Augmented Matrix $\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 0 & 0 & b_2 - b_1 + b_3 \\ -3 & 3 & -3 & b_3 \end{bmatrix}$ $R_1 \times (-1) + \longrightarrow R_2$
$R_3 \times 1 \longmapsto R_2$

$\boxed{-b_1 + b_2 + b_3 = 0}$

20 b) $A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$Ax = 4x$

$\Rightarrow (A - 4I)\,x = 0$

$\Rightarrow \cancel{x \ne 0}$

$$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -2 & 2 \\ 3 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 & 0 \\ 2 & -2 & 2 \\ 3 & 1 & -3 \end{bmatrix} \xrightarrow[R_1 \cdot R_2]{} \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 3 & 1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -2 \\ 3 & 0 & -3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\boxed{\begin{array}{l} \therefore x_2 = 0 \\ \& \ x_1 = x_3 \end{array}}$$

21) $$\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}}_{A} X = \underbrace{\begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}}_{B}$$

$A = 3 \times 3 \quad \& \quad B = 3 \times 5$

$\therefore X = 3 \times 5$

$$X = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}}$$

3

1.7

(2) (a)

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & -1 \\ 4 & 10 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 & 3 \\ 1 & 2 & 0 \\ -5 & 1 & -2 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \cancel{-24}$$

$$\begin{bmatrix} 8 & -2 & 6 \\ -1 & -2 & 0 \\ -20 & 4 & -8 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -24 & -10 & 12 \\ 3 & -10 & 0 \\ 60 & 20 & -16 \end{bmatrix}$$

(4) (a) $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ not ~~symmetric~~ symmetric

(b) $\begin{bmatrix} 3 & 4 \\ 4 & 0 \end{bmatrix}$ symmetric

(c) $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 5 & 1 \\ 3 & 1 & 7 \end{bmatrix}$ symmetric

(d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ not symmetric

(6)

$$A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$$

$a - 2b + 2c = 3$

$2a + b + c = 0$ ~~$\Rightarrow b + c = -2a$~~

$a + c = -2 \Rightarrow c = -2 - a$

$2a + b - 2 - a = 0 \Rightarrow a + b = 2$

$a - 2b - 4 - 2a = 3 \Rightarrow -a - 2b = 7$

$-b = 69$ $b = -\cancel{6}9$

$a = \cancel{8}11$

$c = \cancel{-10}-13$

$\cancel{(a,b,c) : (8, 6, -10)}$

$\boxed{(a,b,c)\ (\cancel{8}11, -9, -13}$

(10) (a)

$$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b)

$$A^{-2} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}} \qquad \cancel{A = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/6 \end{bmatrix}}$$

$$\boxed{A = \begin{bmatrix} \pm 1/3 & 0 & 0 \\ 0 & \pm 1/2 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}}$$

(11) (a)

$$A = \begin{bmatrix} 3a_{11} & 5a_{12} & 7a_{13} \\ 3a_{21} & 5a_{22} & 7a_{23} \\ 3a_{31} & 5a_{32} & 7a_{33} \end{bmatrix}$$

if D is diagonal then

$$A = BD = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\boxed{D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}}$$

(12)

$$A^3 = \begin{bmatrix} 1 & 30 \\ 0 & -8 \end{bmatrix} \qquad A^2 = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{12}(\\ & \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a_{11}^2 & a_{12}(a_{11}+a_{22}) \\ 0 & a_{22}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}^3 & a_{11}^2 a_{12} + a_{11}a_{12}a_{22} + a_{12}a \\ 0 & a_{22}^3 \end{bmatrix}$$

$a_{11}^3 = 1 \Rightarrow a_{11} = 1$

$a_{22}^3 = -8 \Rightarrow a_{22} = -2$

$a_{11}^2 a_{12} + a_{11}a_{12}a_{22} + a_{12}a_{22}^2 = 30 \Rightarrow a_{12} - 2a_{12} + 4a_{12} = 30$

$3a_{12} = 30 \qquad a_{12} = 10.$

$$\therefore A = \begin{bmatrix} 1 & 10 \\ 0 & -2 \end{bmatrix}$$

2.1

$$A = \begin{vmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{vmatrix}$$

(2) (a) (1) m_{13}

$$= (1) \begin{vmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = (1)\left(3(0)\right) - 0$$

$\boxed{m_{13} = 0}$

(2) $C_{13} = (-1)^{1+3} m_{13} = 1 \cdot m_{13} = 0$

$\boxed{C_{13} = 0}$

(4) (a)

$$A = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix}$$

$C_{11} = 29 \qquad C_{12} = -21 \qquad C_{13} = 27$

$C_{21} = 11 \qquad C_{22} = 13 \qquad C_{23} = 5$

$C_{31} = -19 \qquad C_{32} = 19 \qquad C_{33} = 19$

$$\boxed{(\text{adj})\,A = \begin{vmatrix} 29 & 11 & -19 \\ -21 & 13 & 19 \\ 27 & 5 & 19 \end{vmatrix}}$$

(5) $A = \begin{vmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -7 & 0 & 5 \end{vmatrix} = 5\begin{vmatrix} -3 & 7 \\ -7 & 5 \end{vmatrix} = 5(-15+7)$

$\therefore \boxed{\det(A) = -40}$

(16) $7x_1 - 2x_2 = 3$ $\quad A = \begin{vmatrix} 7 & -2 \\ 3 & 1 \end{vmatrix}$ $\quad \det(A) = 13$

$3x_1 + x_2 = 5$

$A_1 = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix}$ $\quad A_2 = \begin{vmatrix} -2 & 3 \\ 1 & 5 \end{vmatrix}$

$\det(A_1) = 26$ $\quad \det(A_2) = -13$

$x_1 = \cancel{2}\ -1 \quad x_2 = \cancel{0}\ 2$

$A_1 = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$ $\quad A_2 = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix}$

$\det(A_1) = 13$ $\quad \det(A_2)\ 26.$

$x_1 = \frac{13}{13} = 1$ $\quad x_2 = \frac{26}{13} = 2$

$\boxed{(x_1, x_2) = (1, 2)}$

(23) Use cramer's rule:

$4x + y + z + w = 6$

$3x + 7y - z + w = 1$

$7x + 3y - 5z + 8w = -3$

$x + y + z + 2w = 3$

$A = \begin{vmatrix} 4 & 1 & 1 & 1 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 1 & 1 & 1 & 2 \end{vmatrix}$

$\det(A) = 4\begin{vmatrix} 7 & -1 & 1 \\ 3 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & -1 & 1 \\ 7 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 7 & 1 \\ 7 & 3 & 8 \\ 1 & 1 & 2 \end{vmatrix} \cancel{-2} - 1\begin{vmatrix} 3 & 7 & -1 \\ 7 & 3 & -5 \\ 1 & 1 & 1 \end{vmatrix}$

$= 4[-126-2+8] - 1[-54-6+12] + 1[-6-42+4] - 1[24-84-4]$

$= 4(-120) - 1(-48) + 1(-44) - 1(-64)$

$= -480 + 48 - 44 + 64$

$\therefore det.(A) = -412$

$$A_y = \begin{vmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & -1 & 1 \\ -3 & -5 & 8 \\ 3 & 1 & 2 \end{vmatrix} - 6\begin{vmatrix} 3 & -1 & 1 \\ 7 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 & 1 \\ 7 & -3 & 8 \\ 1 & 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 & -1 \\ 7 & -3 & -5 \\ 1 & 3 & 1 \end{vmatrix}$$

$= 4[-18+30+12] - 6[-54-6+12] + 1[-90-6+24] + 1[-36-12-24]$

$= 4(24) - 6(-48) + 1(-72) - 1(-72)$

$= 96 + 288 \cancel{-72} \cancel{+72}$

$det(A_y) = 384$

$y = -\frac{384}{412} = -\frac{192}{206} = -\frac{96}{103}$

1.6	06
1.7	11
2.1	65