

Math 534A Practice Test 1

1. Prove that  $x_n = 3 + (-1)^n/n!$  has a convergent subsequence by exhibiting one, finding its limit and proving directly from the  $\epsilon - N$  definition of limit that the subsequence converges.

2. Consider the equivalence relation  $\simeq$  defined on  $\mathbb{R}$  as follows:

$$x \simeq y \text{ iff } \exists k \in \mathbb{Z} \subset \mathbb{R} \text{ such that } y - x = 2k\pi$$

a. Show that  $\simeq$  is an equivalence relation on  $\mathbb{R}$ .

b. Choose a definition for canonical representative for each element of the quotient set  $\mathbb{R}/\simeq$  and show that each equivalence class has exactly one such element.

3. Prove that if  $x_n \rightarrow x$  and  $y_n \rightarrow y$  then  $x_n y_n \rightarrow xy$ .