

Math 534A Problem Set on Quotient Sets

1. Given the set $S = \{a, b, c, d, e\}$ and an equivalence relation \cong such that $a \cong e$, $b \cong d$, and $d \cong c$, find the quotient set S/\cong .
2. Let \mathcal{N} be the set of natural numbers, which for the purposes of this problem start at 0. Define the relation \cong on pairs of natural numbers by $(a, b) \cong (c, d)$ iff $a + d = b + c$.
 - a. Show that this defines an equivalence relation on $\mathcal{N} \times \mathcal{N}$.
 - b. Show that for every equivalence class $\text{mod } \cong$, there is exactly one $a \in \mathcal{N}$ such that the equivalence class has an element of the form $(a, 0)$ or $(0, a)$.
 - c. Define $\mathcal{Z} = \mathcal{N} \times \mathcal{N}/\cong$. Identify non-negative integers with equivalence classes with an element of the form $(a, 0)$ and non-positive integers with those that have an element of the form $(0, a)$. Show that addition defined $\mathcal{N} \times \mathcal{N}$ by $(a, b) + (c, d) = (a + c, b + d)$, is well defined on \mathcal{Z} by showing that if $(a, b) \cong (x, y)$ and $(c, d) \cong (u, v)$ then $(a + c, b + d) \cong (x + u, y + v)$, i.e. show that if we use any elements of the equivalence classes then we get equivalent results.