1.39. Definition. The extended real number system consists of the real number system to which two symbols, $+\infty$ and $-\infty$, have been adjoined, with the following properties:

(*a*) If x is real, then $-\infty < x < +\infty$, and

$$x + \infty = +\infty, \qquad x - \infty = -\infty, \qquad \frac{x}{+\infty} = \frac{x}{-\infty} = 0.$$

(*b*) If $x > 0$, then

$$x \cdot (+\infty) = +\infty, \qquad x \cdot (-\infty) = -\infty.$$

(*c*) If $x < 0$, then

$$x \cdot (+\infty) = -\infty, \qquad x \cdot (-\infty) = +\infty.$$

When it is desired to make the distinction between real numbers on the one hand and the symbols $+\infty$ and $-\infty$ on the other hand quite explicit, the former are called finite.

1.40. Definition. Let E be a set whose members are in the extended real number system. If E is not bounded above (i.e., if for every real y there is an $x \in E$ such that $y < x$), we define $+\infty$ to be the lub of E.

Similarly, the glb of a set E which is not bounded below is defined to be $-\infty$.

Thus, in the extended real number system, every set has a lub and a