

1. Find the critical points, if any, of the function

$$f(x,y) = x^3 + y^3 - 3x - 12y + 25$$

and determine their nature.

2. What is the minimum value of $f(x,y) = \frac{3}{x} + 5xy + \frac{2}{y}$, for positive values of x and y ?

3a. A parabola of the form

$$f(x) = a x + b x^2$$

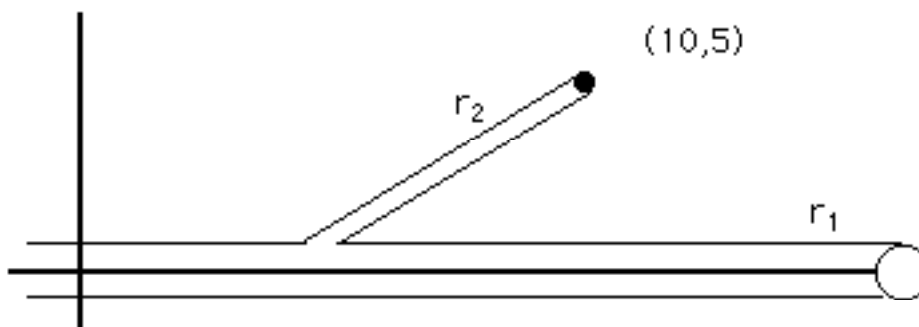
is to be fit to the points $\{(x_i, y_i), i=1 \dots n\}$. Find a formula for the values of a and b which minimize the sum of the squared deviations between the y_i 's and the values $f(x_i)$.

3b. Use your formula to find a and b for the points $(0,0)$, $(1,1)$ and $(-1,2)$.

4. A surgeon is faced with the problem of grafting an artery. She wishes to minimize the resistance to the resulting flow. Resistance R to laminar flow in a pipe is given by Poiseuille's law to be

$$R = L / r^4$$

where L and r are the length and radius of the pipe. The graft must run from a main artery of radius r_1 to a point 5 cm. from the main artery, using a connecting artery of radius r_2 . Coordinatize the problem by assuming that the main artery runs along the x -axis and the connecting artery needs to run to the point $(10,5)$.



Write the total resistance from the origin to the point $(10,5)$ assuming that the graft occurs at $(x,0)$ and write a necessary condition for x to minimize the resistance. Solve your condition for the optimal angle in terms of the radii..