

A mining company owns two mines. Both produce a mixture of high grade (HG), medium grade (MG) and low grade (LG) ore. Mine 1 costs \$200/hr to operate and produces 6 tons of HG, 2 tons of MG, and 4 tons of LG per hour. Mine 2 costs \$160/hr to operate and produces 2 tons of HG, 2 tons of MG, and 12 tons of LG. The company needs to fill orders for 12 tons of HG, 8 tons of MG, and 24 tons of LG.

Formulate the problem of filling the orders with minimum cost. Write the Kuhn-Tucker conditions for this problem.

Recall the Kuhn-Tucker conditions for a minimum problem:

Problem: Min  $f_0(x)$ , with  $f_i(x) \geq 0$ ,  $i=1, \dots, m$

K-T Conditions:

optimality	$\nabla f_0 = \sum_{i=1}^m \lambda_i \nabla f_i$
feasibility	$f_i \geq 0, i=1, \dots, m$
dual feasibility	$\lambda_i \geq 0, i=1, \dots, m$
complimentary	
slackness	$\sum_i \lambda_i f_i = 0$

Define a “vertex of the problem to be where any 2 of the 5 inequality constraints for this problem hold with equality. At each of the ten vertices, find  $\lambda_i$  that make the optimality condition and the complimentary slackness conditions hold. Note that this is accomplished at that vertex by writing  $\nabla f_0$  as a linear combination of the active constraints at the vertex. For each vertex, check primal and dual feasibility and show which (if any) fails.