

1. Solve the following problem using the Kuhn-Tucker conditions and verify your solution geometrically

$$\begin{aligned} &\text{Find the extrema of } x \\ &\text{with } (x-4)^2 + y^2 \leq 16 \\ &\text{and } (x-3)^2 + (y-2)^2 = 13 \end{aligned}$$

2. Write down the Kuhn-Tucker conditions for the problem

$$f(\mathbf{X}) = f(x,y,z) = x^2 + y^2 + z^2$$

subject to

$$g_1(\mathbf{X}) = g_1(x,y,z) = -x + y - z \geq -10$$

and

$$g_2(\mathbf{X}) = g_2(x,y,z) = x + y + 4z \geq 20$$

Obtain the solution in any manner.