1. Find all critical points of the function

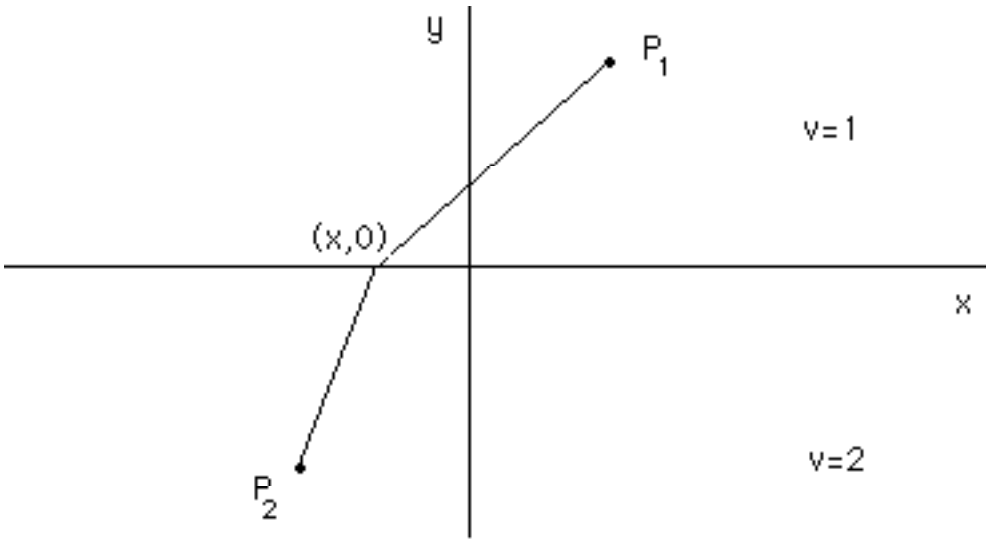
$$f(x,y) = x^6 + y^4 - 6x - 4y + 25$$

and determine their nature.

2. Find the minimum and maximum values of the function

$$f(x,y) = \frac{5x^2 + 3y^2}{x^2 + y^2}$$

for $(x,y) \neq (0,0)$.



3. (Fermat's Principle) A ray of light is to move from the point $P_1=(1, 2)$ in the (x, y) plane to the point $P_2=(-1, -2)$. In the half plane $y>0$, the velocity of light is 1 and in the half plane $y<0$ the velocity of light is 2. Write an expression for the time required by the ray of light to reach (-1, -2) assuming that it crosses the x-axis at (x, 0). Write a necessary condition for x to minimize the time. Interpret your condition geometrically.

4. Find the values of A, B, and C which minimize the sum squared error in a functional fit of the form

$$f(x) = A\cos(x) + B\exp(Cx)$$

to the data

x	y
-2	0.3
-1	3
0	5
1	6
2	7

5. Show that (1,1,1) is a critical point of the function

$$f(X) = f(x,y,z) = x^2 + 2y^2 + 4z^2 + 5xy$$

subject to

$$g_1(X) = g_1(x,y,z) = x + y^2 + 3yz - 5 = 0$$

and

$$g_2(X) = g_2(x,y,z) = x^2 + 5xy + 3z^2 - 9 = 0$$

and determine its nature.

6a. In a production process utilizing three inputs X, Y, and Z, the quantity produced $Q(x,y,z)=xy^3z^2$, where x, y, and z are the number of units of X, Y, and Z respectively.

6b. If the price of a unit of X is $ 5, the price of a unit of Y is $ 2, and the price of a unit of Z is $3, what is the maximum product that can be produced with a budget of B=$100?

6c. What is the value of the derivative dQ_{max}/dB? How is it related to $\partial Q/\partial X$ at the maximum?