February 16, 2007. **Math 121A** Name _____

Instructor: _____ Lab Section time _____

Midterm #2
THEORETICAL EXAM with answers

PRACTICE VERSION of the index card and simple scientific calculators only portion.

1. **(20 pts)** Given that $\exp(a) = 2, \exp(b) = 3, \ln(c) = 6$, and $\ln(d) = 4$, find
   - $\frac{e^0+e^a}{e^b} = 1$
   - $e^{a+b} = 6$
   - $e^{3a+b} = 24$
   - $\ln(\sqrt{d}) = 2$
   - $\ln(c/d) = 2$

2. **(10 pts)** The volume of an animal is 300 cm^3. What is the volume of a similarly shaped animal that is twice the height?
   ANSWER: 2400cm^3.

3. **(10 pts)** The lifetime of an erythrocyte is allometrically related to the weight of the animal. If $T = 3 * w^{1/2}$, find k and r such that $w = k * T^r$.
   ANSWER: $k = 1/9$ and $w = 2$.

4. **(20 pts)** The population of Botsylvania is 10 million as measured by the 1990 census and 20 million as measured by the 2000 census. Assuming that the population follows a Malthusian growth law, $P_{n+1} = (1 + r)P_n$, where n is in years, find the *annual* growth rate r for Botsylvania over this time period.
   ANSWER: $r = 0.07177$

5. **(20 pts)** Given the following table of weights of a culture taken at the recorded times, which hour shows the largest growth rate?

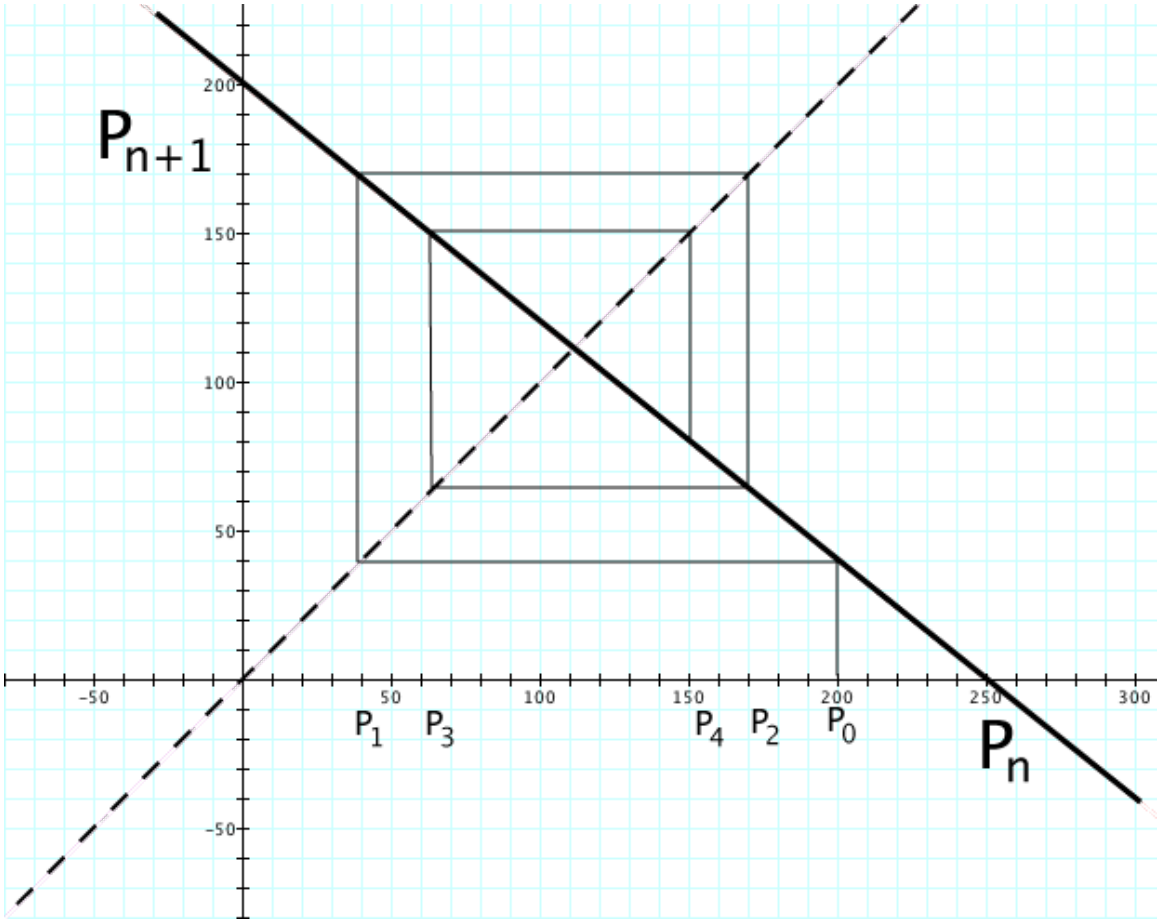
t in hours	weight in grams
0	6.03
2	7.00
3	7.43
4	7.75
7	8.15

ANSWER: The first hours show the largest growth rate.

6. **(20 pts)** A population evolves according to the linear growth law including immigration

$$P_{n+1} = -0.8P_n + 200.$$

The graph below shows a graph of this update rule along with the $y = x$ line. Draw at least four steps of the cobweb diagram for this population starting from $P_0 = 200$.



Midterm #2
LABORATORY EXAM with answers

1. **(40 pts)** The data you downloaded shows the population of a colony of dingbats as a function of n in years. Assume that the population follows a linear growth law
$P_{n+1} = (1 + r)P_n + m$,

- Use EXCEL's trendline to find r and m to 4 significant figures.

  ANSWER: $r = 0.03123, m = 32.06$

- Predict the population for $t = 25$.

  TWO REASONABLE CORRECT ANSWERS: Predicting from P_0 gives $P_{25} = 1619$, predicting from the last population known (P_{18}) gives $P_{25} = 1702$.

- Find the equilibrium population size?

  ANSWER: The equilibrium population size is -1026 (oops).

- Is it stable? How do you know?

  ANSWER: It is unstable as may be see from starting the population near the equilibrium value and noting that it moves away from equilibrium. Alternatively, it is unstable since the slope in the recurrence relation is larger than one.

2. **(30 pts)** The second dataset in your download shows the population of Mauritania as a function of n in decades. Assume that the population follows a non-autonomous growth law

$$P_{n+1} = (1 + k(n))P_n$$

where n is in years. Assume that k(n) is a linear function of n and use EXCELS trendline on the annual values of $P_{n+1}/P_n - 1$ to find m and b such that

$$k(n) = m * n + b.$$

ANSWER: $m = 0.20$ and $b = 0.031$.

3. **(30 pts)** A population is growing according to the non-autonomous growth law

$$P_{n+1} = (1 + k(n))P_n$$

where n is in hours and

$$k(n) = \begin{cases} 0.1 & \text{if } n \leq 1 \\ 0.05 & \text{if } n > 1 \end{cases}$$

When does this population reach twice its initial size?

CORRECTION: This should have read "At which hour does the population first exceed twice its initial size?".

ANSWER: It first reaches a population greater than twice the initial during the 13-th hour (between 12 hours and 13 hours after the start of the experiment).