Math 241 – MATLAB
10/30/06; Due 11/6/06

Math 241 Final Problem

The following problem requires the solution of a system of differential equations and then feeding some parameters based on the result to an optimization routine.

A toy cannon shoots pellets. In a certain contest, a target 10 meters from the cannon (both target and cannon on the ground, y=0) must be hit. What is the largest mass m that can hit the target? What should the angle α equal to achieve this hit? The equations of motion and the initial conditions are given below.

$$\begin{bmatrix} \dot{x} \\ \dot{v}_x \\ \dot{y} \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} v_x \\ (-.5 - .1v_x)/m \\ v_y \\ -9.8 \end{bmatrix} \qquad \begin{bmatrix} x(0) \\ v_x(0) \\ y(0) \\ v_y(0) \end{bmatrix} = 10 \begin{bmatrix} 0 \\ \cos(\alpha) \\ 0 \\ \sin(\alpha) \end{bmatrix} / \sqrt{m}$$

Hint: The key to making successful MATLAB applications is to build a function, make a script to run it, turn that script into a function, make a script to run it, turn that script into a function, …

One way to proceed is to build a function to evaluate the derivative (as a 4-tuple) and write a script that assigns values of m and α and finds the trajectory using ode45. Extracting x and y from the solution enables you to evaluate x_{final} using

```
y_of_x=@(xx) spline(x,y,xx)
fzero(y_of_x, x0)    % x0 is a starting guess for how far it goes
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Then if you turn this script into a function receiving [m, α] as input and returning $(x_{final} - 10)^2 - m^2$ as output, you can send this function to fminsearch to find the largest m that will hit (x,y)=(10,0).

Turn in a cleaned up version of your calculations, any programs that you used, and write some conclusions in a well-written sentence or two. In addition, turn in a graph illustrating your results.

If you cannot get all the way to the answer, carry out the directions in the paragraphs above as far as you can.