Math 241-MAPLE Final Exam

1. A current research problem (pursued in Anca Segal's lab, SDSU biology) concerns protein binding. It involves looking at a system of three algebraic equations

$$\frac{R_1}{R} = \frac{x}{K_1 + x} \tag{1}$$

$$\frac{R_2}{R} = \frac{nx}{K_2 + x} \tag{2}$$

$$x = L - R_1 - R_2. \tag{3}$$

The parameters R, K_1, K_2, and n represent constants. In the experiment, the value of L is adjusted and the value of R_1 is measured. Use the three equations above to eliminate x and R_2, thereby obtaining an equation between L and R_1 involving only constants. Formally:

a. Derive an expression for L in terms of R_1 and the constants R, K_1, K_2, and n but not involving the other variables x and R_2.

For the next two parts, use the values $R = 1, n = 10, K_1 = 2, K_2 = 3$.
b. Plot L as a function of R_1 in the interval $R_1 = 0..0.9$.
c. What value of R_1 will give an L value of 10?

2. When fired, a toy cannon imparts a kinetic energy $E = mv^2/2$ to a small pellet that is to hit a target on the floor L meters from the cannon and which is also on the floor. There is a strong fan blowing exactly away from the target with the result that the equations of motion for the x and y coordinates of the pellet are

$$\frac{d^2x}{dt^2} = -w, \tag{4}$$

$$\frac{d^2y}{dt^2} = -9.8, \tag{5}$$

with w being a constant related to the strength of the wind created by the fan. If the cannon is aimed at an angle θ to the floor, the initial conditions for the velocity are $V_{0x} = v\cos(\theta), V_{0y} = v\sin(\theta)$.

a. Find the distance traversed by the pellet as a function of E, m, w, and θ. Note that this is given by the change in x during the time y takes to return to 0.

For the next four parts, use the values $E = 1, w = 3$.
b. Find the value of θ that maximizes the distance traversed.
c. Find the largest distance traversed as a function of m.
d. Find the largest mass $m_{\max}$ that can hit the target a distance $L = 1$ away.
e. Plot a curve showing the trajectory of the pellet.
EXTRA CREDIT: Create an animation showing the flight of the pellet.

3. Write a procedure that, given an integer q, returns the integer p such that p/q is as close as possible to 225/157.
EXTRA CREDIT: Solve the current Problem of the Fortnight.