Math 241E Practice Final - Linear Algebra

1. Solve for $(x_1, x_2, x_3)$ that satisfy the following system of equations

$$\begin{aligned} 3.4x_1 + 2.8x_2 + 3.1x_3 &= 4 \\ 2.1x_1 + x_2 + 9.1x_3 &= 6.2 \\ 4.3x_1 + 8.7x_3 &= 2.1 \end{aligned}$$

2. For which value of $a$ does the linear system with coefficient matrix

$$\begin{pmatrix} a & 0 & 9 \\ 9 & 4 & 4 \\ 0 & 2 & 1 \end{pmatrix}$$

fail to have a unique solution?

3. Find all values of $k$ that make $\mathbf{u} = (k, 2, 5, 2)$ and $\mathbf{v} = (6, k, 0, 2 - k)$ orthogonal.

4. Express the vector $\mathbf{u} = (3, 5, 8)$ relative to the basis $S\{(1, 1, 0), (1, -1, 0), (0, 0, 1)\}$.

$(\mathbf{u})_S =$

5. Find the determinant of the following matrix

$$A = \begin{pmatrix} 1.2 & 2.1 & 3.7 & -12.4 & 15.3 \\ 1.1 & 2.3 & 4.9 & 12 & 0.1 \\ 0 & 0 & 1.4 & 4.5 & 5.6 \\ 3.3 & 6 & 10.9 & 30 & 0 \\ 1.7 & 2.3 & 3 & 9 & 0.7 \end{pmatrix}$$

6. Find the transpose of the matrix in problem 5. (Use EXCEL's TRANSPOSE function.)

7. Express the vector $\mathbf{u} = (3, 5, 8)$ relative to the basis $S = \{(1, 1, 0), (1, -1, 0), (0, 0, 1)\}$.

$(\mathbf{u})_S =$

8. A random walk such as the one you simulated in homework 1 can be represented by a matrix. Consider a random walk on the numbers 1 to n and define the matrix $T$ whose $(i,j)^{th}$ entry is the probability that if the process is in state $i$ at time $t$, then it will be in state $j$ at time $t+1$. $T$ is called the transition matrix of the random walk.

Consider a random walk on the set $\{1, 2, ..., n\}$ with absorbing boundary conditions and with $p$ being the probability of moving left and $q = 1 - p$ being the probability of moving to the right at any of the points sctrictly between 1 and n. For $n = 5$ this has transition probability matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ p & 0 & 1-p & 0 & 0 \\ 0 & p & 0 & 1-p & 0 \\ 0 & 0 & p & 0 & 1-p \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The advantage of the matrix representation is that the power $T^k$ of $T$ describe what happens in $k$ steps, i.e., $(T^k)_{i,j}$ is the probability that the walk starting in state $i$ at time 0 is in state $j$ at time $k$. Thus by finding large powers of $T$, we can calculate what happens in the long run.

a. What is the probability of reaching 1 starting from 4 when n=7 and p=0.1?

b. What is the probability of reaching 1 starting from 4 when n=7 and p=0.4?

9. (Simulation) Given that $x_i, i = 1, 2, 3$ are independent and uniform $[0, 1]$ random variables, find the average value of $y = x_1^2 + x_2^2 + x_3^2$.

Extra Credit: Draw the distribution of $y$.