

1. Find all values of k that make $\mathbf{u} = (k, 2, 0, 2 - k)$ and $\mathbf{v} = (6, k, k^3, 2 - k)$ orthogonal.
 Ans. $k = -2$.

2. Find the standard matrix for the following composition of linear transformations that map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$:

A reflection across the x -axis followed by a 45° rotation followed by another reflection across the x -axis.

Ans. $\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$.

3. A linear transformation $\mathbf{T} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ sends \mathbf{e}_1 to $2\mathbf{e}_1 - \mathbf{e}_2$ and \mathbf{e}_2 to $-3\mathbf{e}_1$. What is the standard matrix of \mathbf{T} ?

Ans. $\begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix}$.

4. Consider the vector space $\mathbb{P}(3)$ of cubic polynomials and the linear transformation $D : \mathbb{P}(3) \rightarrow \mathbb{P}(3)$, which sends any polynomial to its derivative,

$$D(a + bx + cx^2 + dx^3) = b + 2cx + 3dx^2.$$

Using the standard basis $\{1, x, x^2, x^3\}$ in $\mathbb{P}(3)$, find the standard matrix of the linear transformation D .

Ans. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

5. Find a basis for $\text{span}(\{(2, 2, 7), (5, 5, 0), (-2, -2, 6)\})$.

Ans. $\{(2, 2, 7), (5, 5, 0)\}$ (or many other possibilities).

6. A 10 by 4 matrix M has rank 3. Find the nullity of M and M^T .

Ans. $\text{nullity}(M) = 1$ and $\text{nullity}(M^T) = 7$.

7. Find the rank and nullity of

$$\begin{pmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{pmatrix}$$

Ans. Rank = 3, Nullity = 2.

8. Find bases for the row space and column space of the matrix in the previous problem.

Ans. A basis for the row space is $S = \{1, 0, 8, 2, -2\}, (0, 1, 2, 0, -1), (0, 0, 1, 0, -5/12)$. A basis

for the column space is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -3 \\ 3 \\ -3 \\ -6 \\ -9 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ -2 \\ 0 \\ 2 \end{pmatrix} \right\}$.

9. Find a basis for the null space of the matrix in the previous two problems.

Ans. A basis for the nullspace is $\left\{ \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4/3 \\ 1/6 \\ 5/12 \\ 0 \\ 1 \end{pmatrix} \right\}$.

10. Express the vector $\mathbf{u} = (3, 5, 7)$ relative to the basis $S\{(1, 3, 5), (0, 0, 1), (1, 1, 1)\}$.

$$(\mathbf{u})_S = (1, 0, 2)_S.$$