Section	2.2	2.3	3.1	3.2	3.3	3.4	3.5
Total questions.	06	09	06	04	03	03	04

$\boxed{\text{Total} \quad x/35}$ ①

2.2

(2a) Evaluate from inspection

$$\begin{vmatrix} 3 & -17 & 4 \\ 0 & 5 & 1 \\ 0 & 0 & -2 \end{vmatrix} \Rightarrow \cancel{30} \; 3 \times 5 \times (-2)$$

$$\boxed{\det(A) = -30}$$

(3a)

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \Rightarrow 1 \begin{vmatrix} 1 & 0 & 0 \\ 0 & -5 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1 \times 1 \times (-5) \times 1$$

$$\boxed{\det(A) = -5}$$

(5)

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} \qquad \det(A) = \begin{vmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 3 & 2 & 4 \end{vmatrix}$$

$(R_1 \leftrightarrows$

$$-\begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & -1 & -2 \end{vmatrix} R_3 \leftarrow R_1 \times (-3) \;\rightarrow\; -\begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & -5/3 \end{vmatrix} R_3 \leftarrow R_2 (1/3)$$

$$\rightarrow \frac{1}{3}\begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{vmatrix} \begin{matrix} \text{common factor} \\ -1/3 \end{matrix} \quad = \frac{1}{3}(1 \times 3 \times 5) \quad = 5$$

$$\boxed{\det(A) = 5}$$

(8)

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix} \qquad \det(A) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & -1 \end{vmatrix} \begin{matrix} R_2 \leftarrow R_1 \times (-5) \\ R_3 \leftarrow R_1 \times (1) \\ R_4 \leftarrow R_1 \times (-2) \\ \end{matrix} \rightarrow \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 108 & 23 \end{vmatrix} R_4 \leftarrow R_2(-12)$$

$$\rightarrow \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & -13 \end{vmatrix} R_4 \leftarrow R_3(36) \quad = 1 \times 1 \times (-3) \times (-13) = +39$$

$$\boxed{\det(A) = +39}$$

(12)

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$$

(b)

$$\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & f \\ 4g & 4h & 4i \end{vmatrix} = (-12) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-12) \times (-6) = 72$$

(3 from R_1, -1 from R_2, 4 from R_3)

$$\boxed{\det(A) = 72}$$

(d)

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix} = -3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} R_3 \leftarrow R_2(-4)$$

$$= (-3) \times (-6) = 18$$

$$\boxed{\det(A) = 18}$$

2.3

(5) $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $\det(A) = -7$.

(a) $\det(3A)$

$\det(kA) = k^n(\det(A))$ if A is $n \times n$

$\det(3A) = 3^3(-7) = 27 \times (-7) = -189$

$\boxed{\det(3A) = -189}$

(b) $\det(A^{-1}) = \dfrac{1}{\det(A)} = -\frac{1}{7}$

$\boxed{\det(A^{-1}) = -1/7}$

(c) $\det(2A^{-1}) = 2^3 \times \dfrac{1}{\det(A)} = \dfrac{8}{-7} = -8/7$.

$\boxed{\det(2A^{-1}) = -8/7}$

(14) $(\lambda I - A)x = 0$

(a) $\left.\begin{aligned} x_1 + 2x_2 &= \lambda x_1 \\ 2x_1 + x_2 &= \lambda x_2 \end{aligned}\right\} \Rightarrow$ From $Ax = \lambda x$ $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\boxed{\begin{bmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$

(b) $\begin{aligned} 2x_1 + 3x_2 &= \lambda x_1 \\ 4x_1 + 3x_2 &= \lambda x_2 \end{aligned} \Rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore \boxed{\begin{bmatrix} \lambda-2 & -3 \\ -4 & \lambda-3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$

(15) (i) Characteristic Equation.

(a) $(\lambda I - A) = 0$.

$$\begin{vmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{vmatrix} = 0 \quad \text{OR} \quad (\lambda-1)^2 - 4 = 0$$

(b) $$\begin{vmatrix} \lambda-2 & -3 \\ -4 & \lambda-3 \end{vmatrix} = 0 \quad \text{OR} \quad (\lambda-2)(\lambda-3) - 12 = 0$$

$$\lambda^2 - 5\lambda - 6 = 0.$$

(ii) Eigen Values:

(a) $(\lambda-1)^2 = 4$

$\lambda - 1 = \pm 2$

$\boxed{\lambda = 3 \text{ or } \lambda = -1}$

(b) $(\lambda-2)(\lambda-3) - 12 = 0$

$\Rightarrow \lambda^2 - 5\lambda - 6 = 0$

$\Rightarrow \lambda^2 - 6\lambda + \lambda - 6 = 0$

$\Rightarrow \lambda(\lambda-6) + 1(\lambda-6) = 0$

$\Rightarrow (\lambda-6)(\lambda+1) = 0$

$\boxed{\lambda = 6 \text{ or } \lambda = -1}$

3.1

(3a) Component of the vectors. initial point P_1 terminal

$P_1(4,8)$, $P_2(3,7)$

$\overrightarrow{P_1P_2} = (3-4, 7-8) = (-1,-1)$

$\boxed{\overrightarrow{P_1P_2} = (-1,-1)}$

(4a) nonzero vector u. initial point $P(-1,3,-5)$ such that u has the same direction as $v = (6,7,-3)$

~~vector. when u & v have the vectors having same direction~~

$u = K(6,7,-3)$ direction vector = $(6,7,-3)$

$\therefore \boxed{v = (-1,3,-5) + t(6,7,-3)}$

(6b) $u = (-3,1,2)$, $v = (4,0,-8)$, $w = (6,-1,-4)$

$6u + 2v = (-18, 6, 12) + (8, 0, -16)$

$\boxed{6u + 2v = (-10, 6, -4)}$

(10) Find scalars c_1, c_2 & c_3 such that

$c_1(1,2,0) + c_2(2,1,1) + c_3(0,3,1) = (0,0,0)$

$c_1 + 2c_2 + 0c_3 = 0$ —— (i)

$2c_1 + c_2 + 3c_3 = 0$ —— (ii)

$0c_1 + c_2 + c_3 = 0$ —— (iii)

(i)×(-2)+(ii) $\quad -3c_2 + 3c_3 = 0$

& $c_2 + c_3 = 0$

$\therefore \boxed{c_1 = c_2 = c_3 = 0}$

(11)(a) $P(2,3,-2)$ & $Q(7,-4,1)$

midpoint of segment connecting PQ

$\left(\frac{7+2}{2}, \frac{-4+3}{2}, \frac{1+(-2)}{2}\right) = \boxed{\cancel{\left(\frac{5}{2}, \frac{-7}{2}, \frac{3}{2}\right)}}$

$\boxed{(9/2, -1/2, -1/2)}$

(13) $P(1,3,7)$ $x(4,0,-6)$ is Midpoint connecting P & Q

$$(4,0,-6) = \left(\frac{x+1}{2}, \frac{y+3}{2}, \frac{z+7}{2}\right)$$

$x+1 = 8$ $\quad y+3 = 0$ $\quad z+7 = -12$

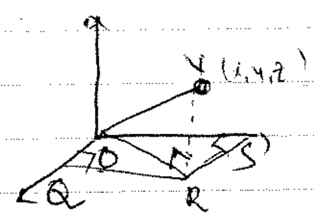
$x = 7$ $\quad y = -3$ $\quad z = -19$

$\cancel{\boxed{Q(9,3,-5)}}$ $\quad \boxed{Q = (7,-3,-19)}$

3.2

1) Norm of v

(d) $\vec{v} = (2,2,2)$



$$\|v\| = \sqrt{(2-0)^2+(2-0)^2+(2-0)^2} = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{}$$

$$\boxed{\|v\| = 2\sqrt{3}}$$

2(a) Distance between P_1 & P_2

$P_1(3,4)$, $P_2(5,7)$

$\overrightarrow{P_1P_2} = (2,3)$

$|P_1P_2| = \sqrt{4+9} = \sqrt{13}$.

$$\boxed{|P_1P_2| = \sqrt{13}}$$

5(a) $u = (2,0,4)$ & $v = (1,3,-6)$

$ku + lv = (5,9,-14)$

$k(2,0,4) + l(1,3,-6) = (5,9,-14)$

$2k+l=5$
$3l=9 \Rightarrow l=3$
$4k-6l=-14$

$\therefore 2k+3=5 \Rightarrow 2k=2 \Rightarrow k=1$

$\boxed{k=1 \ \& \ l=3 \text{ such that } ku+lv=(5,9,-14)}$

(9)(b) if v is non zero vector then $\frac{1}{\|v\|}v$ is a unit vector.
Use this result to find vector that has the same direction as the vector $(3,4)$

$\|v\| = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore (u_1, u_2) = (3/5, 4/5)$.

$\therefore$ Unit vector in direction of $(3,4) = (3/5, 4/5)$

3.3:

(2a) $u=(2,3)$, $v=(5,-7)$.
find the cosine of the angle θ between u & v
If θ is the angle between u & v then

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|} = \frac{(2,3)\cdot(5,-7)}{\sqrt{4+9}\cdot\sqrt{25+49}}$$

$$= \frac{10-21}{\sqrt{13}\times\sqrt{74}} = \frac{-11}{\sqrt{74\times13}} = \frac{-11}{31.016}$$

$\therefore \boxed{\cos\theta \approx -0.355}$

(16) (a) $p = (2, k)$ & $q = (3, 5)$ find k such that

(a) p & q are parallel.

if p & q are parallel then

$$p = m q$$

$\therefore 2 = m \cdot 3$ & $k = m \cdot 5$

$\Rightarrow m = \frac{2}{3}$ $\Rightarrow$ $k = \frac{2}{3} \times 5$ $\therefore k$

$$\boxed{k = 10/3}$$

(b) p & q are orthogonal.

$p \cdot q = 6 + 5k$

For orthogonal solutions $p \cdot q = 0$

$\therefore 6 + 5k = 0$

$\Rightarrow \boxed{k = -6/5}$

3.4

1 (a) $u = (3, 2, -1)$, $v = (0, 2, -3)$ & $w = (2, 6, 7)$

(a) $v \times w$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ 2 & 6 & 7 \end{vmatrix}$$

$= (14 + 18) i - (6) j + (-4) k$

$= 22i - 6j - 4k$

$$\therefore \boxed{(v \times w) = (32, -6, -4)}$$

(9) If $u \cdot (v \times w) = 3$ Find

(a) $u \cdot (w \times v)$

$$w \times v = \begin{vmatrix} i & j & k \\ w_1 & w_2 & w_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \quad \& \quad v \times w = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Interchanging 2^{nd} & 3^{rd} row we can get one from another.

$\therefore (v \times w) = -(w \times v)$

$\Rightarrow u \cdot (v \times w) = -u(w \times v)$

$\Rightarrow u(w \times v) = -3$

$$\boxed{u \cdot (w \times v) = -3}$$

(11) (b) $u = (5, -2, 1)$, $v = (4, -1, 1)$ & $w = (1, -1, 0)$

Determine weather u, v & w lie in the same plane ~~where~~ when positioned so that their initial points coincide.

taking determinent value of these 3 points

$$\begin{vmatrix} 5 & -2 & 1 \\ 4 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 5(0+1) - (-2)(0-1) + 1(-4+1)$$
$$= 5 - 2 - 3$$
$$= 0$$

The absolute value of the determinant is zero. So, ~~we~~ the volume of the parallelepiped whose sides are the vectors u, v & w is equal to 0.

$\therefore$ **u, v & w lie in the same plane.**

3.5.

(1) (a) $P(-1, 3, -2)$; $n = (-2, 1, -1)$

Find a ~~normal~~ point-normal form of the equation of the plane passing through P & having n as a normal.

For given P & n, point normal form is given by

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$(-2)(x - (-1)) + (1)(y - 3) + (-1)(z - (-2)) = 0$$

$$\therefore \boxed{-2(x+1) + (y-3) - (z+2) = 0}$$

(8) Determine whether the line and plane are perpendicular

(b) $x = 2 + t$, $y = 1 - t$, $z = 5 + 3t$; $6x + 6y - 7 = 0$

$(x, y, z) = (2, 1, 5) + t(1, -1, 3)$

which is parallel to vector $v(1, -1, 3)$

plane ~~line~~ $6x + 6y + 0z = 7$

$n\ (6, 6, 0)$

here we are not getting any ~~...~~ relationship between n & v. Such that

$n \neq kv$ for any $k \in R$

$\therefore$ $\boxed{\text{Line \& plane are not perpendicular}}$

(10) Find parametric equation for the line passing through the given points.

(a) $(5, -2, 4)$, $(7, 2, -4)$

We are seeking paramatric equation for line l which ~~...~~ is passing through both points P_1 & P_2

$\overrightarrow{P_1P_2} = (2, 4, -8)$

Now selecting first point (any one from the given)

$(x-5, y+2, z-4) = t(2, 4, -8)$

$\therefore$ $\boxed{x = 5+2t,\ y = -2+4t,\ z = 4-8t}$ $(-\infty < t < \infty)$

(21) Find an equation for the plane that passes through the point $(3, -6, 7)$ & parallel to the plane $5x - 2y + z - 5 =$

$\Rightarrow$ Parallel planes have same normal

$\therefore$ Normal to plane $5x - 2y + z - 5 = 0$ is $n = (5, -2, 1$

Now ~~taking eqn to find~~ considering the given point

$(5, -2, 1)(x-3, y+6, z-7) = 0$

$5x - 15 - 2y - 12 + z - 7 = 0$

$\boxed{5x - 2y + z - 34 = 0}$