[redacted]

4.2	4.3	4.4	Total
7	9	5	21

4.2

(3) Find the standard matrix of the linear operator $T: R^3 \to R^3$ given by

$$w_1 = 3x_1 + 5x_2 - x_3$$
$$w_2 = 4x_1 - x_2 + x_3$$
$$w_3 = 3x_1 + 2x_2 - x_3$$

and calculate $T(-1,2,4)$ by directly substituting in the equations and also by matrix multiplication

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$(x_1, x_2, x_3) = (-1, 2, 4)$

$$w_1 = 3(-1) + 5(2) - 1(4) = -3 + 10 - 4 = 3$$
$$w_2 = 4(-1) - 1(2) + 1(4) = -4 - 2 + 4 = -2$$
$$w_3 = 3(-1) + 2(2) - 1(4) = -3 + 4 - 4 = -3$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

$$\boxed{T(-1,2,4) = (3,-2,-3)}$$

(4a) Find the standard for the linear operator T defined by the formula.

$$T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$$
$$w_1 = 2x_1 - x_2$$
$$w_2 = x_1 + x_2$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}$$

(8c) Use matrix multiplication to find reflection of $(-1, 2)$ about:

the line $y = x$

standard matrix for the linear operator T reflecting each vector about the line $y = x$ in R^2

$$[T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$w = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\boxed{T(-1, 2) = (2, -1)}$$

(11a) Use matrix multiplication algorithm to find orthogonal projection of $(-2, 1, 3)$ on <u>xy plane</u>

xy plane in R^3

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

w of $(-2, 1, 3)$

$$w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{T(-2, 1, 3) = (-2, 1, 0)}$$

(12a) Use matrix multiplication to find image of the vector (3, −4) when it is rotated through an angle of $\theta = 30^\circ$

Angle θ in (counterclockwise) in R^2 is

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$w = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3\sqrt{3}}{2} + \frac{4}{2} \\ \frac{3}{2} - \frac{4\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{3}+4}{2} \\ \frac{3-4\sqrt{3}}{2} \end{bmatrix}$$

$$\boxed{T(3,-4) = \left(\frac{3\sqrt{3}+4}{2} , \frac{3-4\sqrt{3}}{2} \right)}$$

(13a) Use matrix multiplication to find the image of vector (−2, 1, 2) if it is rotated 30° about the x-axis.

θ about the x-axis (counterclockwise) in R^3 is

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{\sqrt{3}}{2} - 1 \\ \frac{1}{2} + \sqrt{3} \end{bmatrix}$$

$$\boxed{T(-2,1,2) = \left(-2, \frac{\sqrt{3}-2}{2}, \frac{1+2\sqrt{3}}{2}\right)}$$

(19a) Find the standard matrix for the stated composition of linear operators on R^3

A rotation of $30°$ about the x-axis, followed by a rotation of $30°$ about the z-axis, followed by contraction with factor $k = 1/4$.

$$[T_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30° & -\sin 30° \\ 0 & \sin 30° & \cos 30° \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$\theta = 30°$ about z-axis in R^3 as

$$[T_2] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 30° & -\sin 30° & 0 \\ \sin 30° & \cos 30° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

T_3, contracting each vector with factor $k = 1/4$ in R^3 is

$$[T_3] = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$$[T] = [T_3][T_2][T_1] \quad \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 1 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & - \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\boxed{T = \begin{bmatrix} \frac{\sqrt{3}}{8} & -\frac{\sqrt{3}}{16} & \frac{1}{16} \\ \frac{1}{8} & \frac{3}{16} & -\frac{\sqrt{3}}{16} \\ 0 & \frac{1}{8} & \frac{\sqrt{3}}{8} \end{bmatrix}}$$

4.3

(2) Find the standard matrix for linear ~~operation~~ operator defined by the equations, and use Theorem 4.3.4 to determine weather the operator is one to one.

(a) $w_1 = 8x_1 + 4x_2$

$w_2 = 2x_1 + x_2$

matrix ~~form~~ form of these equations

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & 4 \\ 2 & 1 \end{bmatrix}$$

$\det[A] = 8 - 8 = 0$

$\therefore$ $\boxed{\text{We have, that linear operator } T_A \text{ is not one to one}}$

(b) $w_1 = 2x_1 - 3x_2$

$w_2 = 5x_1 + x_2$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix}$$

$\det[A] = 4 + 15 = 19 \neq 0$

$\boxed{T_A \text{ is one to one}}$

(5a) Determine weather the linear operator $T : R^2 \to R^2$ defined by the equations is one to one; if so, fi[nd] the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2)$

$$w_1 = x_1 + 2x_2$$
$$w_2 = -x_1 + x_2$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$[T^{-1}] = [T]^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$[T^{-1}]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{w_1 - 2w_2}{3} \\ \frac{w_1 + w_2}{3} \end{bmatrix}$$

$$\boxed{T^{-1}(w_1, w_2) = \left(\tfrac{1}{3}w_1 - \tfrac{2}{3}w_2,\ \tfrac{1}{3}w_1 + \tfrac{1}{3}w_2\right)}$$

(13a) Use theorem 4.3.3 to find the standard matrix for $T: R^2 \rightarrow R^2$ from the images of the standard basic vectors.

(a) $T: R^2 \rightarrow R^2$ projects a vector orthogonally on to the x-axis and then reflects the vector about y axis.

- Any vector $x = (x, y)$ transforms to the vector $(-x, 0)$

$$T(e_1) = \begin{bmatrix}-1\\0\end{bmatrix}, \quad T(e_2) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\boxed{[T] = \begin{bmatrix}-1 & 0\\0 & 0\end{bmatrix}}$$

(b) $T: R^2 \to R^2$ reflects a vector about line $y = x$ and then reflects the vector about x-axis.

$$T(e_1) = \begin{bmatrix}0\\-1\end{bmatrix} \quad T(e_2) = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\therefore \quad \boxed{[T] = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}}$$

(14) Use Theorem 4.3.3 to find the standard matrix for $T: R^3 \to R^3$ from the images of the standard basic vectors

(a) $T: R^3 \to R^3$ reflects a vector about the xz plane and then contracts that vector by factor of $1/5$

Any vector $x = (x, y, z)$ transforms to $\left(\frac{x}{5}, \frac{-y}{5}, \frac{z}{5}\right)$

$$T(e_1) = \begin{bmatrix}1/5\\0\\0\end{bmatrix}, \quad T(e_2) = \begin{bmatrix}0\\-1/5\\0\end{bmatrix}, \quad T(e_3) = \begin{bmatrix}0\\0\\1/5\end{bmatrix}$$

$$\boxed{[T] = \begin{bmatrix}1/5 & 0 & 0\\0 & -1/5 & 0\\0 & 0 & 1/5\end{bmatrix}}$$

(c)(b) $T: R^3 \to R^3$ projects a vector orthogonally onto ~~xz~~ xy plane ~~and then projects that vector orthogonally onto the xy-plane~~ reflects the vector about xz-plane, and then reflects the vector about yz-plane

$T(e_i)$ $x = (x, y, z)$ transforms to $(x, -y, -z)$

$$T(e_1) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad T(e_2) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \; \& \; T(e_3) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\boxed{[T] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

(15) Let $T_A : R^3 \to R^3$ be multiplication by

$$A = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 2 \\ 4 & 5 & -3 \end{bmatrix}$$

and let $e1, e2$ and $e3$ be the standard basis vectors for R^3. Find the following vectors by inspection.

(a) $TA(e_1)$, $TA(e_2)$ & $TA(e_3)$

$$TA(e_1) = TA\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 2 \\ 4 & 5 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \underline{\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}}$$

$$\underline{TA(e_2) = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}} \quad \& \quad \underline{TA(e3) = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}}$$

(b) $T_A(e1 + e2 + e3)$

$$= TA\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 2 \\ 4 & 5 & -3 \end{bmatrix}$$

$$\boxed{T_A(e_1 + e_2 + e_3) = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}}$$

[redacted]

<u>4.4</u>

(9a) Identify the operators on polynomial that corresponds to the following operation on vectors. Give resulting polynomial

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

Let $p(x) = x^2 + 2x - 1$

& $q(x) = 3x^2 + 2$

$$-2\begin{bmatrix} \cancel{6}3 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ -4 \end{bmatrix}$$

~~8.[q(x)] = 8.3x² + 8.2 = 24x² + 16~~

~~vector~~ $-2 \cdot q(x) = -6x^2 - 4$

then vector sum

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} -6 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ -5 \end{bmatrix}$$

$$\boxed{p(x) - 2q(x) = -5x^2 + 2x - 5}$$

(9b) Consider the following matrix. What is corresponding transformation on polynomials? Indicate the domain P_i and co-domain P_j

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ a+b \\ 2a-b \end{bmatrix}$$

$$\boxed{ax^2 + (a+b)x + (2a-b)}$$

[redacted]

(9a) Consider the space of all functions of the form $a+bt+ce^{t}+de^{-t}$ where a,b,c,d are real scalars. What function in the space corresponds to the sum of $(1,2,3,4)$ and $(-1,-2,0,-1)$ assuming that we represent a function in this space as the vector (a,b,c,d)?

$$z_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \& \quad z_2 = \begin{bmatrix} -1 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$z_1 + z_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 3 \end{pmatrix}$$

corresponding function

$$\boxed{3e^{t} + 3e^{-t}}$$

(13)
(a) Find a quadratic interpolant to the data $(-2,1)$, $(0,1)$, $(1,4)$ using the Vandermonde system approach form 1.

$$\begin{bmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

$\therefore$ For given data

$$\begin{bmatrix} 1 & -2 & 4 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

So Solution is

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\boxed{P(x) = x^2 + 2x + 1}$$

(b) Repeat using the Newton approach form 4 to

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & x_1 - x_0 & 0 \\ 1 & x_2 - x_0 & (x_2 - x_0)(x_2 - x_1) \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

∴ For given data

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$b_0 = 1$$
$$b_0 + 2b_1 = 1 \Rightarrow b_1 = 0$$
$$b_0 + 3b_1 + 3b_2 = 4 \Rightarrow b_2 = 1$$

$$\boxed{\begin{aligned} P(x) &= (x+2)(x-0) + 1 \\ &\text{or } = (x+1)^2 \\ P(x) &= x^2 + 2x + 1 \end{aligned}}$$