5.1	5.2	Total
08	19	27

1A

5.1:

Axioms:

If the following axioms are satisfied by all objects, u, v, w in V and all scalars k and m, then we call V a vector space and we call objects in V vectors.

1) If u and v are objects in V, then $u+v$ is in V
2) $u+v = v+u$
3) $u+(v+w) = (u+v)+w$
4) There is an object 0 in V, called a zero vector for V, such that $0+u = u+0 = u$ for all u in V
5) For each u in V, there is an object $-u$ in V, called a negative of u, such that $u+(-u) = (-u)+u = 0$
6) If k is any scaler and u is any object in V, then ku is in V.
7) $k(u+v) = ku+kv$
8) $(k+m)u = ku+mu$
9) $k(mu) = (km)u$
10) $1u = u$

Determine which sets are vector spaces under the given operations. For those that are not vector spaces, list all axioms that fail to hold.

1) The set of all triples of real numbers (x, y, z) with the operations
$(x, y, z) + (x', y', z') = (x+x', y+y', z+z')$ and $k(x, y, z) = (kx, y$
let $u = (x, y, z)$ & $v = (x', y', z')$
Applying all axioms
1) $u+v = (x+x', y+y', z+z')$
2) $u+v = (x+x', y+y', z+z') = (x'+x, y'+y, z'+z) = v+u$
3) $u+(v+w) = (x, y, z) + (x'+x'', y'+y'', z'+z'')$

$= (x+x'+x'', y+y'+y'', z+z'+z'')$
$= (x+x', y+y', z+z') + (x'', y'', z'') = (u+v)+w$

4) vector $(0,0,0)$

5) $u=(x,y,z)$, triple $(-x,-y,-z)$ is obviously the object $-u$ and satisfies axiom 5.

6) $ku = k(x,y,z) = (kx, y, z)$

7) $k(u+v) = k(x+x', y+y', z+z') = (k(x+x'), y+y', z+z')$
$= (kx, y, z) + (kx', y', z') = ku+kv$

8) let $k=1$ & $m=1$
$(k+m)(x,y,z) = 2(x,y,z) = (2x, y, z)$
& $k(x,y,z) + m(x,y,z) = 1(x,y,z) + 1(x,y,z) = (2x, 2y, 2z)$
X 8 Fails

9) $k(mu) = k(mx, y, z) = (kmx, y, z) = (km)(x,y,z)$

10) $1 \cdot u = (1 \cdot x, y, z) = (x,y,z) = u$

$\boxed{\text{V is not a vector space, because Axiom 8 fails}}$

2) The set of all triples of real numbers (x,y,z) with the operations
$(x,y,z) + (x',y',z') = (x+x', y+y', z+z')$ and $k(x,y,z) = (0,0,0)$

let $u=(x,y,z)$ & $v=(x',y',z')$

Applying all axioms

1) $u+v = (x+x', y+y', z+z')$

2) $u+v = (x+x', y+y', z+z') = (x'+x, y'+y, z'+z) = v+u$

3) $u+(v+w) = (x,y,z) + (x'+x'', y'+y'', z'+z'') = (x+x', y+y', z+z') + (x'', y'', z'')$
$= (u+v)+w$

4) Triple $(0,0,0)$

5) For each $u=(x,y,z)$ the triple $(-x,-y,-z)$ is obviously the object $-u$ and satisfies Axiom 5

6) Scalar multiplication operation is not the standard

mutiplication

$ku = k(x,y,z) = (0,0,0)$

7) $k(u+v) = k(x+x', y+y', z+z') = (0,0,0)$

$ku+kv = k(x,y,z) + k(x',y',z') = (0,0,0)+(0,0,0) = (0,0,0)$

8) $(k+m)u = (k+m)(x,y,z) = (0,0,0)$

9) $k(m(u)) = k(m(x,y,z)) = k(0,0,0) = (0,0,0)$

$(km(u)) = (km)(x,y,z) = (0,0,0)$

10) $1 \cdot u = 1\ (x,y,z) = (0,0,0) \neq u$ 10 Fails

$\therefore$ $\boxed{\text{V is not a vector space, Axiom 10 Fails}}$

3) The set of all pairs of real numbers (x,y) with the operations

$(x,y) + (x',y') = (x+x', y+y')$ & $k(x,y) = (2kx, 2ky)$

let $u = (x,y)$ & $v = (x',y')$

1) $u+v = (x,y) + (x',y') = (x+x', y+y')$

2) $u+v = (x+x', y+y') = (x'+x, y'+y) = v+u$

3) $u+(v+w) = (x,y) + (x'+x'', y'+y'') = (x+x'+x'', y+y'+y'')$

$= (x+x', y+y') + (x'', y'') = (u+v)+w$

4) Pair $(0,0)$

5) for each $u=(x,y)$, pair $(-x,-y)$ is obviously the object $-u$

6) $ku = k(x,y) = (2kx, 2ky)$

7) $k(u+v) = k(x+x', y+y') = (2k(x+x'), 2k(y+y'))$

$= (2kx+2kx', 2ky+2ky') = (2k(x,y)) + (2k(x',y')) = ku+kv$

8) $(k+m)u = (k+m)(x,y) = (2(k+m)x, 2(k+m)y)$

$= (2kx+2mx, 2ky+2my) = (2kx, 2ky) + (2mx, 2my) = ku+mu$

9) $k(mu) = k(m(x,y)) = k(2mx, 2my) = (4kmx, 4kmy)$

$(km)u = (2kmx, 2kmy)$

$\therefore k(mu) \neq (km)u$ 9 fails

10) $1u = 1(x,y) = (2x, 2y) \neq (x,y) = u$ 10 Fails

$\therefore$ $\boxed{\text{V is not a vector space, Axiom 9 \& 10 Fails}}$

2B

4) The set of all real numbers x with the standard operations of addition & multiplication

Let $u = x$ & $v = x'$

1) $u+v = x+x'$

2) $u+v = x+x' = x'+x = v+u$

3) $u+(v+w) = x+(y+z) = x+y+z = (x+y)+z = (u+v)+w$

4) Numbers $= \emptyset$

5) For each $u = x$, number $-x$

6) $ku = kx$

7) $k(u+v) = k(x+y) = kx+ky = ku+kv$

8) $(k+m)u = (k+m)x = kx+mx = ku+mu$

9) $k(mu) = k(mx) = kmx = (km)u$

10) $1u = 1x = x = u$

All axioms holds

$\therefore$ $\boxed{V \text{ is a vector } \cancel{\text{in}} \text{ space.}}$

4) The set of all n-tuples of real numbers of the form $(x, x, \ldots, x)$ with the standard operations on R^n

Let $u = (x, x, \ldots, x)$ & $v = (x', x', \ldots, x')$

1) $u+v = (x, x, \ldots, x) + (x', x', \ldots, x') = (x+x', x+x', \ldots, x+x')$

2) $u+v = (x+x', x+x', \ldots, x+x') = (x'+x, x'+x, \ldots, x'+x) = v+u$

3) $u+(v+w) = (x, x, \ldots, x) + (x'+x'', x'+x'', \ldots, x'+x'') = (x+x'+x'', x+x'+x'', \ldots$

$= (x+x', x+x', \ldots, x+x') + (x'', x'', \ldots, x'') = (u+v)+w$

4) n-tupple $(\emptyset, \emptyset, \ldots, \emptyset)$

5) For each $u = (x, x, \ldots, x)$ n-tuple $(-x, -x, \ldots, -x)$

6) $ku = k(x, x, \ldots, x) = (kx, kx, \ldots, kx')$

7) $k(u+v) = k((x, x, \ldots, x) + (x', x', \ldots, x')) = k(x+x', x+x', \ldots, x+x')$

$= (k(x+x'), k(x+x'), \ldots, k(x+x')) = (kx+kx', kx+kx', \ldots, kx+kx')$

$= (kx, kx, \ldots, kx) + (kx', kx', \ldots, kx') = k(x, x, \ldots, x) + k(x', x', \ldots, x') = ku+kv$

8) $(k+m)u = (k+m)(x, x, \ldots, x) = ((k+m)x, (k+m)x, \ldots, (k+m)x)$

$= (kx+mx, kx+mx, \ldots, kx+mx) = (kx, kx, \ldots, kx) + (mx, mx, \ldots, mx)$

$= k(x, x, \dots, x) + m(x, x, \dots, x) = ku + mu$

9) $k(mu) = k(m(x, x, \dots, x)) = k(mx, mx, \dots, mx) = (kmx, kmx, \dots, km$
$= (km)(x, x, \dots, x) = (km)u$

10) $1u = 1(x, x, \dots, x) = (x, x, \dots, x) = u$

$\therefore$ All axioms holds

$\therefore$ $\boxed{V \text{ is a vector space}}$

9) The set of all 2×2 matrices of the form
$$\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$$
with the standard matrix addition & scalar multiplicatio

Let $u = \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ & $v = \begin{bmatrix} a' & 1 \\ 1 & b' \end{bmatrix}$.

1) $u+v = \begin{bmatrix} a+a' & 2 \\ 2 & b+b' \end{bmatrix}$ Axiom 1 Fails.

2) $u+v = \begin{bmatrix} a+a' & 2 \\ 2 & b+b' \end{bmatrix} = \begin{bmatrix} a' & 1 \\ 1 & b' \end{bmatrix} + \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = v+u$

3) $u+(v+w) = \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} a'+a'' & 2 \\ 2 & b'+b'' \end{bmatrix} = \begin{bmatrix} a+a'+a'' & 3 \\ 3 & b+b'+b'' \end{bmatrix}$

$= \begin{bmatrix} a+a' & 2 \\ 2 & b+b' \end{bmatrix} \begin{bmatrix} a'' & 1 \\ 1 & b'' \end{bmatrix} = (u+v)+w$

4) Doesn't have zero Axiom 4 Fails

5) No such negative object. Axiom 5 Fails.

6) For any real number $k \neq 1$ Axiom 6 Fails.

$ku = k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ak & k \\ k & bk \end{bmatrix}$

7) $k(u+v) = k\begin{bmatrix} a+a' & 1+1 \\ 1+1 & b+b' \end{bmatrix} = \begin{bmatrix} k(a+a') & k(1+1) \\ k(1+1) & k(b+b') \end{bmatrix}$

$= \begin{bmatrix} ka+ka' & k+k \\ k+k & kb+kb' \end{bmatrix} = ku + kv$

8) $(k+m)u = \begin{bmatrix} (k+m)a & (k+m)1 \\ (k+m)1 & (k+m)b \end{bmatrix} = \begin{bmatrix} ka+ma & k+m \\ k+m & kb+mb \end{bmatrix}$

$= ku + mu.$

9) $k(mu) = k\begin{bmatrix} ma & m \\ m & mb \end{bmatrix} = \begin{bmatrix} kma & km \\ km & kmb \end{bmatrix}$

$= km\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = (km)u$

10) $1u = 1\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = u.$

$\boxed{\text{V is not a vector space, axioms 1,4,5 \& 6 fails}}$

13) The set of all pairs of real numbers of the form $(1, x)$ with the operations

$(1, y) + (1, y') = (1, y+y')$ & $k(1, y) = (1, ky)$

Let $u = (1, y)$ & $v = (1, y')$

1) $u+v = (1, y) + (1+y') = (1, y+y')$

2) $u+v = (1, y) + (1+y') = (1, y'+y) = (1, y') + (1, y) = v+u$

3) $u+(v+w) = (1, y) + (1, y'+y'') = (1, y+y'+y'')$

$= (1, y+y') + (1, y'') = (u+v)+w$

4) Pair $(1, 0)$

5) For $u = (1, y)$, the pair $(1, -y)$

6) Any real k

$\therefore k(1, y) = (1, ky)$

7) $k(u+v) = k((1, y) + (1, y')) = k(1, y+y') = (1, k(y+y'))$

$= (1, ky+ky') = \cancel{k}(1, ky) + (1, ky') = ku + kv$

8) $(k+m)u = (k+m)(1+y) = (1, (k+m)y) = (1, ky+my)$

$= (1, ky) + (1, my) = ku + mu$

9) $k(mu) = k(1, my) = (1, kmy) = km(1, y) = (km)u$

10) $1 \cdot u = 1 \cdot (1, y) = (1, 1 \cdot y) = (1, y) = u.$

$\therefore$ All axioms holds

$\therefore$ $\boxed{\text{V is a vector space.}}$

15) The set of all positive real numbers with the operations $x+y = xy$ & $kx = x^k$

let $u = x$ & $v = y$

1) $u+v = x+y = xy$
2) $u+v = x+y = xy = yx = y+x = v+u$
3) $u+(v+w) = x(yz) = (xy)z = (u+v)+w$
4) $\emptyset = x^{\emptyset} = 1$
5) For each $u = x$ $\quad -u = x^{-1}$ is object o $-u$
6) For any real no. k, $ku = kx$
7) $k(u+v) = k(x+y) = k(xy) = (xy)^k = x^k y^k = (kx)(ky)$
$= kx + ky = ku + kv$
8) $(k+m)u = (k+m)x = x^{k+m} = x^k x^m = (kx)(mx)$
$= ku + mu$
9) $k(mu) = k(mx) = kx^m = (x^m)^k = x^{mk} = x^{km}$
$= (km)x = (km)u$
10) $1 \cdot u = x^1 = x = u$.

$\therefore$ All axioms holds

$\therefore$ $\boxed{V \text{ is vector space}}$

5.2

1) Use theorem 5.2.1 to determine which of the following are subspaces of R3

(a) all vectors of the form $(a, \emptyset, \emptyset)$

let $u = (a_1, \emptyset, \emptyset)$ & $v = (a_2, \emptyset, \emptyset)$

$u+v = (a_1+a_2, \emptyset, \emptyset) \in W$

$ku = (ka_1, \emptyset, \emptyset) \in W$

$\therefore \in R3$

(b) All vectors of the form $(a, 1, 1)$

let $u = (a_1, 1, 1)$ & $v = (a_2, 1, 1)$

$u+v = (a_1+a_2, 2, 2) \notin W$

$\therefore \notin R3$

(c) All vectors of the form (a, b, c) where $b = a+c$

Let $u = (a_1, a_1+c_1, c_1)$ & $v = (a_2, a_2+c_2, c_2)$

$u+v = (a_1+a_2, a_1+a_2+c_1+c_2, c_1+c_2) \in W$

$ku = (ka_1, ka_1+kc_1, kc_1) \in W$

$\therefore \in R^3$

(d) All vectors of the form (a, b, c) where $b = a+c+1$

Let $u = (a_1, a_1+c_1+1, c_1)$, & $v = (a_2, a_2+c_2+1, c_2)$

$u+v = (a_1+a_2, a_1+a_2+c_1+c_2+2, c_1+c_2) \notin W$

$\notin R^3$

(e) All vectors of the form $(a, b, \emptyset)$

Let $u = (a_1, b_1, \emptyset)$ & $v = (a_2, b_2, \emptyset)$

$u+v = (a_1+a_2, b_1+b_2, \emptyset) \in W$

$ku = (ka_1, kb_1, \emptyset) \in W$

$\in R^3$

$\therefore$ $\boxed{\begin{matrix} a, c, e \in R^3 \\ b, d \notin R^3 \end{matrix}}$

7) Which of the following are linear combinations of $u = (\emptyset, -2, 2)$ & $v = (1, 3, -1)$?

(a) $(2, 2, 2)$

Let us denote the target vector as P. In order for P to be a linear combination of u and v, there must be scalars k_1 & k_2 such that

$P = k_1 u + k_2 v$

$\therefore (2, 2, 2) = k_1(\emptyset, -2, 2) + k_2(1, 3, -1)$

$\therefore (2, 2, 2) = (k_2, -2k_1+3k_2, 2k_1-k_2)$

$$\left.\begin{aligned} k_2 &= 2 \\ -2k_1+3k_2 &= 2 \\ 2k_1-k_2 &= 2 \end{aligned}\right\} \quad k_1 = k_2 = 2$$

Such k_1 & k_2 exist.

$\therefore$ $\boxed{\text{Linear combination of } u \,\&\, v}$

(b) $(3,1,5)$

$(3,1,5) = k_1(0,-2,2) + k_2(1,3,-1)$

$(3,1,5) = (k_2, -2k_1+3k_2, 2k_1-k_2)$

$$\left.\begin{aligned} k_2 &= 3 \\ -2k_1+3k_2 &= 1 \\ 2k_1-k_2 &= 5 \end{aligned}\right\} \text{ no such } k_1 \,\&\, k_2 \text{ exist.}$$

$\therefore$ $\boxed{\text{Not a linear combination of } u \,\&\, v}$

(c) $(0,4,5)$

$(0,4,5) = k_1(0,-2,2) + k_2(1,3,-1)$

$$\left.\begin{aligned} k_2 &= 0 \\ -2k_1+3k_2 &= 4 \\ 2k_1-k_2 &= 5 \end{aligned}\right\} \text{ no such } k_1 \,\&\, k_2 \text{ exist}$$

$\therefore$ $\boxed{\text{Not a linear combination of } u \,\&\, v}$

(d) $(0,0,0)$

$(\phi,\phi,\phi) = k_1(0,-2,2) + k_2(1,3,-1)$

$$\left.\begin{aligned} k_2 &= \phi \\ -2k_1+3k_2 &= \phi \\ 2k_1-k_2 &= \phi \end{aligned}\right\} \quad k_1 = k_2 = \phi$$

Such $k_1 \,\&\, k_2$ exist.

$\therefore$ $\boxed{\text{Linear combination of } u \,\&\, v}$

$$\boxed{\begin{array}{l}\text{Linear combination of } u \,\&\, v: \text{(a) \& (d)} \\ \text{Not linear combination of } u \,\&\, v: \text{(b) \& (c)}\end{array}}$$

9) Express the following as linear combinations of
$p1 = 2+x+4x^2$, $p2 = 1-x+3x^2$ & $p3 = 3+2x+5x^2$

(a) $-9-7x-15x^2$

5B

Let us write the polynomials in vector form & denote the target vector as p.
In order for p to be a linear combination of u & v, there must be scalars k_1, k_2 and k_3 such that

$$p = k_1 p_1 + k_2 p_2 + k_3 p_3 \quad \text{that is}$$

$$(-9,-7,-15) = k_1(2,1,4) + k_2(1,-1,3) + k_3(3,2,5)$$

$\therefore$

$$2k_1 + k_2 + 3k_3 = -9$$
$$k_1 - k_2 + 2k_3 = -7$$
$$4k_1 + 3k_2 + 5k_3 = -15$$

Solving this system using

$$k_1 = -2,\ k_2 = 1 \ \& \ k_3 = -2$$

$$\therefore \boxed{p = -2p_1 + p_2 - 2p_3}$$

(b) $6 + 11x + 6x^2$

$$(6,11,6) = k_1(2,1,4) + k_2(1,-1,3) + k_3(3,2,5)$$

$\therefore$

$$2k_1 + k_2 + 3k_3 = 6$$
$$k_1 - k_2 + 2k_3 = 11$$
$$4k_1 + 3k_2 + 5k_3 = 6$$

Solving this system

$$k_1 = 4,\ k_2 = -5,\ k_3 = 1$$

$$\therefore \boxed{p = 4p_1 - 5p_2 + p_3}$$

(c) $\emptyset$

$$(0,0,0) = k_1(2,1,4) + k_2(1,-1,3) + k_3(3,2,5)$$

$2k_1 + k_2 + 3k_3 = \emptyset$
$k_1 - k_2 + 2k_3 = \emptyset$
$4k_1 + 3k_2 + 5k_3 = \emptyset$

Solving this system.

$k_1 = k_2 = k_3 = \emptyset$

$\boxed{p = \emptyset}$

(d) $7 + 8x + 9x^2$

$(7, 8, 9) = k_1(2,1,4) + k_2(1,-1,3) + k_3(3,2,5)$
$2k_1 + k_2 + 3k_3 = 7$
$k_1 - k_2 + 2k_3 = 8$
$4k_1 + 3k_2 + 5k_3 = 9$

Solving this system

$k_1 = \emptyset, \quad k_2 = -2 \quad \& \quad k_3 = 3$

$\boxed{p = -2p_2 + 3p_3}$

(13) Determine the following polynomials span P_2.

$p_1 = 1 - x + 2x^2$
$p_2 = 3 + x$
$p_3 = 5 - x + 4x^2$
$p_4 = -2 - 2x + 2x^2$

→ By defination if $S = \{v_1, v_2, \ldots v_r\}$ is a set of vectors in a vector space V, then the subspace ω of V consisting of all linear combinations of the vectors in S is called the space spanned by $v_1, v_2, \ldots, v_r$ and we say that the vectors $v_1, v_2, \ldots, v_r$ span ω

→ Let us determine the dimension of the subspace

spanned by the given vectors. ~~We fi~~
span R^3

Can we write any quadratic polynomial (any 3 tuple) as a linear combination of these vectors.

Looking at a refference of the matrix with these vectors as rows.

these gives

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & -3/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

→ This has only 2 non-zero rows and thus these vectors can only span a 2 space.

$\boxed{\text{These vectors can only span 2 space.}}$

(14) Let $v_1 = (2,1,0,3)$, $v_2 = (3,-1,5,2)$ & $v_3 = (-1,0,2,1)$
Which of the following vectors in span $\{v_1, v_2, v_3\}$

(a) $(2,3,-7,3)$

$$p = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$(2,3,-7,3) = k_1(2,1,0,3) + k_2(3,-1,5,2) + k_3(-1,0,2,1)$$

$\therefore$

$$\begin{aligned} 2k_1 + 3k_2 - k_3 &= 2 \\ k_1 - k_2 &= 3 \\ 5k_2 + 2k_3 &= -7 \\ 3k_1 + 2k_2 + k_3 &= 3 \end{aligned}$$

Solving this system,

$k_1 = 2$, $k_2 = -1$, $k_3 = -1$

$\therefore$ $\boxed{\text{vector is in span}}$

(b) $(\emptyset, \emptyset, \emptyset, \emptyset)$

$$p = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$(\emptyset,\emptyset,\emptyset,\emptyset) = k_1(2,1,\emptyset,3) + k_2(3,-1,5,2) + k_3(-1,\emptyset,2,1)$$

$$2k_1 + 3k_2 - k_3 = \emptyset$$
$$k_1 - k_2 = \emptyset$$
$$5k_2 + 2k_3 = \emptyset$$
$$3k_1 + 2k_2 + k_3 = \emptyset$$

Solving this system.

$k_1 = k_2 = k_3 = \emptyset$

$\therefore$ $\boxed{\text{Vector is in span}}$

(c) $(1,1,1,1)$

$$p = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$(1,1,1,1) = k_1(2,1,\emptyset,3) + k_2(3,-1,5,2) + k_3(-1,\emptyset,2,1)$$

$$2k_1 + 3k_2 - k_3 = 1$$
$$k_1 - k_2 = 1$$
$$5k_2 + 2k_3 = 1$$
$$3k_1 + 2k_2 + k_3 = 1$$

Solving this system

No such k_1, k_2 & k_3 which satisfy all the equation.

$\therefore$ $\boxed{\text{Vector is not in span}}$

(d) $(-4, 6, -13, 4)$

$$p = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$(-4,6,-13,4) = k_1(2,1,\emptyset,3) + k_2(3,-1,5,2) + k_3(-1,\emptyset,2,1$$

$$2k_1 + 3k_2 - k_3 = -4$$
$$k_1 - k_2 = 6$$
$$5k_2 + 2k_3 = -13$$
$$3k_1 + 2k_2 + k_3 = 4$$

Solving this system

$k_1 = 3$, $k_2 = -3$, & $k_3 = 1$

$\therefore$ $\boxed{\text{Vector is in the span}}$

Vector is in the span : (a), (b), (d)
Vector is not in the span : (c)

(15) Find an equation for the line spanned by the vectors $u = (-1, 1, 1)$ & $v = (3, 4, 4)$

By defination

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 3 & 4 & 4 \end{vmatrix}$$

$$= i(\emptyset) - j(-7) + k(-7) = \{0, 7, -7\}$$

$$\boxed{\emptyset x + 7y - 7z = \emptyset}$$