

1. A certain population is assumed to be growing according to an affine model according to

$$P(n+1) = a * P(n) + b$$

- a. Given the data in the online file “PracticeFinalData.xls”. Download the file and find the values of a and b that give the best fit to the data using the initial population $P(0)$ in the dataset.

$$a = \underline{132.7}$$

$$b = \underline{-29.5}$$

- b. What is the predicted equilibrium size of the population? Is the equilibrium stable? **90.3 unstable**
- c. Find the R^2 value measuring goodness of fit for the model in part a.
 $R^2 = \underline{0.873}$

2. A certain age structured population evolves according to the Leslie matrix in the online file “PracticeFinalData.xls”. Note that the file also includes the initial population for each age cohort.

- a. How old does a female member of this population have to be before bearing young? **Over 1 yrs old.**
- b. What fraction of the babies born during the third year have mothers between 2 and three years old? **0**
- c. What is the eventual growth rate of the population? **19.84%**
- d. Predict the total female population for 15 years (years 0 through 15). Use EXCEL’s trendline function to fit an exponential model

$$P(n) = A \exp(B * n)$$

to the total population sizes and report the resulting values below.

$$A = \underline{218.05}$$

$$B = \underline{0.1986}$$

$$R^2 = \underline{0.9302}$$

- e. Compare the value of B obtained and the growth rate you found in part c. How should these be related?
 $\exp(B)=1.219$ should be close to $(1+\text{the rate in c})=1.1986$

3. A certain population grows according to the logistic recurrence relation

$$x(n+1) = x(n) + 1.2x(n) \left(1 - \frac{x(n)}{3000} \right).$$

The initial population is $x(0)=100$.

- a. Find the population after 10 steps and after 10^9 steps. **3000.02, 3000**

- b. Find any steady states of this dynamical system and analyze their stability.
 $x=0$ unstable, $x=3000$ stable.

4. Starting from $x(0)=3000$, the population in problem 3 experiences a steady stream of immigration of 1000 individuals each time period, i.e.

$$x(n+1) = x(n) + 1.2x(n)\left(1 - \frac{x(n)}{3000}\right) + 1000.$$

Assuming this situation persists, find the equilibrium population and analyze its stability.
 3679.45 stable.

5. The population model in problem 4 is modified to

$$x(n+1) = x(n) + rx(n)\left(1 - \frac{x(n)}{3000}\right) + 1000$$

with $r = 1.2$ in good years and $r = 1.03$ in bad years. If the probability of any one year being bad is 0.3, find the mean and standard deviation of the population after a long time.
 Mean 3703, Stdev 90

6. A certain supermarket chain is running a lottery in which the customers have to collect three types of coupons to win. One (scratch off type) coupon is given to each customer every time they shop. The coupons are handed out at random according to the probability distribution

Type A coupons	50%
Type B coupons	30%
Type C coupons	20%

A customer wins if they have collected at least one of each of the three types of coupons. At any time, each shopper is in one of the 8 states

1. Having no coupons
2. Having A coupon only
3. Having B coupon only
4. Having C coupon only
5. Having A and B coupons
6. Having A and C coupons
7. Having B and C coupons
8. Having A, B, and C coupons

a. Write the transition matrix for this process modeled as a Markov chain.

0	0	0	0	0	0	0	0
0.5	0.5	0	0	0	0	0	0
0.3	0	0.3	0	0	0	0	0
0.2	0	0	0.2	0	0	0	0
0	0.3	0.5	0	0.8	0	0	0
0	0.2	0	0.5	0	0.7	0	0
0	0	0.2	0.3	0	0	0.5	0
0	0	0	0	0.2	0.3	0.5	1

- b. Starting from state 1, what is the expected number of shopping trips before reaching state 8? **6.65**
- c. If each of 1000 customers start in state 1 and each makes 10 shopping trips before the end of the contest, how many winning prizes should the store expect to have to pay? **864.4**
7. A chi-squared test of a certain model results in a chi-squared value of 66.387 for 51 observations (50 degrees of freedom). Do you reject the model at the 95% confidence level? **No.**