Math 534A

Problems on Mathematical Induction

1. Prove that for all natural numbers n,

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

2. Prove that for all natural numbers n,

$$\sum_{i=1}^{n} 2^{i-1} = 2^n - 1.$$

3. Prove that for all natural numbers n,

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \left(\sum_{i=1}^{n} i\right)^2.$$

4. Prove that for all natural numbers $n > 3$,

$$2^n < n!.$$

5. Prove that if

$$a_1 = 1, \quad a_2 = 1,$$

and for all natural numbers n,

$$a_{n+2} = a_n + a_{n+1}$$

then for all natural numbers n,

$$a_n = \frac{\sqrt{5}}{5}\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^n - \frac{\sqrt{5}}{5}\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)^n.$$