Math 534A Practice Test 2

Please write up solutions to the following three problems on separate sheets of your own paper using one side only and labeling each sheet with your name, the problem number, and numbering the pages for each problem.

Example:

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Jane Doe
page 2 of Problem 2A
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1. Prove that there exists an increasing sequence of distinct integers $n_1, n_2, ..., n_k, ...$ such that $\lim_{k\to\infty} sin(n_k)$ exists.

2. For each of the following sets, find the

- interior
- closure
- boundary
- set of accumulation points

In addition please decide whether the set is

- open
- closed
- compact
- connected

- a. $[a, b]$
- b. $\mathbb{Z}$
- c. $]2, 3]$
- d. $\{x | x = 3 + (-1)^n 5, \text{ for some } n \in \mathbf{N}\}$
- e. $\{1, 3, -2.7\}$
- f. The rationals in [0,1].

- g. $\{(x, y) \in \mathrm{R}^2 \mid |x| + |y| = 1\}$
- h. $\cup_{n=1}^{\infty} [-1, 1/n[$
- i. $\{(x, y) \in \mathrm{R}^2 \mid x^2 + y^2 \leq 1\}$
- j. $\{(x, y) \in \mathrm{R}^2 \mid |x| \leq 1\}$

3. Determine by proof or counterexample the truth of the following statement:

(A is compact in R^2) $\Rightarrow$ ($\mathrm{R}^2 - A$ is connected).