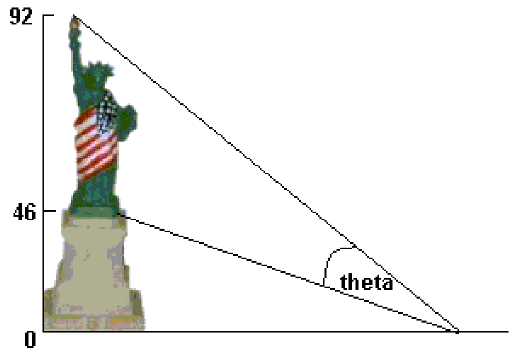
1. The Statue of Liberty stands 92 meters high, including the pedestal which is 46 meters high. How far from the base should you stand so that your viewing angle, theta, is as large as possible? See the figure below.



2. Find the global minimum of the function

$$f(x,y,z)=2x^2+xy+y^2+yz+z^2-6x-7y-8z+9$$

and prove that it is the global minimum.

3. A consumer produces certain goods and services *Y* at home, part of which he consumes. He also works outside the home for an additional wage. Let *y*=units of *Y* which he consumes, *T*=total hours worked, *t*=hours producing *Y*, *M*=money income, *w*=wage earned outside home, *p*=unit price of *Y*. His ability to turn out *Y* is governed by the production function *Y=f(t)*. His total income is then

$$M = p\,[f(t)-y] + w\,(T-t)$$

If his utility depends on *T, M* and *y*, that is, *u=u(T,M,y)*, derive the conditions that *u* be maximum. Show that at optimal *t*,

$$w = p\frac{df}{dt} = -\frac{\dfrac{\partial u}{\partial T}}{\dfrac{\partial u}{\partial M}}$$

and

$$p = \frac{\dfrac{\partial u}{\partial y}}{\dfrac{\partial u}{\partial M}}. \qquad \text{INTERPRET.}$$

4. A system chooses its state so as to maximize

$$f(x,y) = 5x+y \quad \text{subject to } x^2 + y^2 \leq R.$$

If at time t_0, $R=1$ and $dR/dt=2$, what is df_{max}/dt at time t_0?

5. Consider the function $f(x,y)=xy$ in the region $x^2+y^2\leq 1$.

(a) Exhibit all points where the Kuhn-Tucker conditions for f to have a minimum are satisfied.
(b) Exhibit all points where the Kuhn-Tucker conditions for f to have a maximum are satisfied.
(c) Find (with proof) the global minimum and the global maximum.

6. Consider the problem of minimizing the function

$$f(x) = \max \{C_1x, C_2x\} \text{ where } x \in R^n,\ x\geq 0.$$

Transform this to a standard LP of the form min $f=Cy$, $Ay=B$, $y\geq 0$.