

1. Find the area of the largest rectangle with sides parallel to the axes that can be inscribed in the ellipse $x^2 + y^2/4 = 1$.
2. If the sum of two positive numbers is 30, what is the maximum value of their product?
3. Find a necessary condition for the point on the surface $f(x,y,z)=0$ which lies nearest to a given point (x_0, y_0, z_0) . Show that if (x^*, y^*, z^*) satisfies your condition, then the vector from (x^*, y^*, z^*) to (x_0, y_0, z_0) is perpendicular to the surface $f=0$ at (x^*, y^*, z^*) .
4. The sum $S = - \sum_{i=1}^n p_i \ln(p_i)$ is a measure of the amount of randomness in a probability distribution $P = (p_1, p_2, \dots, p_n)$. Find the most random distribution over three categories, i.e., find $P = (p_1, p_2, p_3)$ which maximizes S subject to $\sum_{i=1}^n p_i = 1$.