Math 122   Practice Final

Index cards only. No calculators or notes or computers to be used for this part of the exam.
This part to be turned in before proceeding with the computer portion.

Find the following derivatives:

1. $f(x) = \exp(\frac{\sin(2x)+x}{2})$

$\frac{df}{dx} =$

2. $g(x) = [\cos(3x+2)]^2$

$\frac{dg}{dx} =$

Find the following integrals:

3. $\int \cos(3x+2)\sin(3x+2)\,dx$

4. $\int_{\ln(2)}^{\ln(3)} x\exp(x^2)\,dx$

Solve the following differential equations:

5. $\frac{dy}{dx} = \frac{y}{x};$ $\qquad y(1) = 3.$

6. $\frac{dy}{dt} = -\frac{\sin(t)}{y};$ $\qquad y(0) = 5.$

Full computational aids and open notes portion

1. Find the values of the parameters A, B, omega (ω), and phi (ϕ) that make the fit between the following data

t	f(t)
0	-0.46344
0.1	-0.88371
0.2	-0.90338
0.3	-1.05398
0.4	-0.75755
0.5	-0.45023
0.6	0.00913
0.7	0.60885
0.8	1.23332
0.9	1.85016
1	2.27471
1.1	2.58576
1.2	2.93597
1.3	2.86237
1.4	2.80809
1.5	2.69986
1.6	2.25781
1.7	1.56784
1.8	1.01228
1.9	0.55037
2	-0.03659

and the function

$$f(t) = A + B\sin(\omega t + \phi)$$

as small as possible in the least squares sense.

A = _____

B = _____

ω = _____

ϕ = _____

What are the values of the amplitude, period, and vertical shift for your answer?

2. Find parameters P_0, r and M, that fit the following differential equation

$$\frac{dP}{dt} = -rP(P - M)$$

to the data

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
P	0.032	0.039	0.069	0.11	0.17	0.229	0.261	0.288	0.309	0.327	0.347

3. Find the area of the region bounded by $y = x^2$ and $y = \cos(x)$.

4. A cat jumps from a ledge that is 1 meter off the ground to catch a hummingbird that is hovering x meters above the ground. The highest point of the cat's trajectory is x meters from the ground, i.e. he just manages to catch the bird, and he reaches this highest point one fourth of a second after starting the jump. What is the value of x and how long after starting the jump does the cat reach the ground?

5. Find the absolute maximum value of the function $f(x) = x^2 \exp(-2x)$ for $x > 0$.

6. A certain bacterial population is undergoing Malthusian growth. If its initial mass is 3 grams and three hours later it weighs 6 grams, how much does it weigh 7 hours after the start of the experiment?